Q: I hear people using two terms, square root and radical, to talk about what seems to be the same thing. Is there a difference between these two terms? And what exactly is meant by the term "radical"?

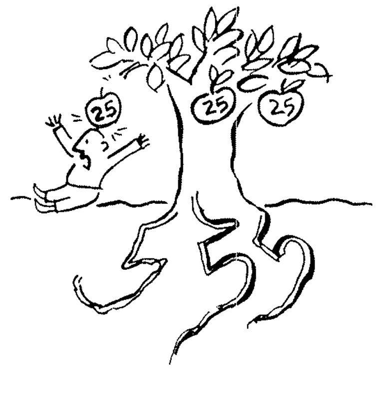
A: Good question. Yes, in higher math there is a distinction between the term **square root** and the term **radical**. But in Algebra I you don't have to worry about this difference, for at this point, the two terms mean the same thing. As to the word **radical**, be aware that it has three shades of meaning. Here they are:

1) Radical means the mere **idea** of a square root, as in the sentence: "This week we're going to study radicals." Translation: This week we're going to study square roots.

2) Radical also means the actual **symbol** for a root, i.e.: $\sqrt{\ }$ So if a teacher points to a term like $\sqrt{\mathbf{5}}$ and asks: "Which term is under (or inside) the radical?" s/he is asking you what is inside the radical sign. In this case, the answer would be the number **5**.

3) Mathematicians use the word **radical** to mean **both the symbol and the number or term under it**. For example, if you write down $\sqrt{\mathbf{25}}$ as the answer to a problem, a teacher might ask you to "simplify the radical." S/he would be asking you to simplify the entire term $\sqrt{\mathbf{25}}$ to **5**.

Info tidbit: We use the word **radical** because it comes from the Latin word **radix**, meaning **root**. The idea seems to be this: just as a root is the **foundation of a tree**, a radical is the **foundation of a number**. For example, since $\mathbf{5} \cdot \mathbf{5} = \mathbf{25}$, you can imagine that the number **25** depends on the number **5** for its existence just as an apple tree depends on a root for its existence.



Q: I think I understand the basic idea of a square root. But what would be the value of a square root times itself? In other words, what would be the value of this expression:

$$\sqrt{a} \cdot \sqrt{a}$$

A: The value of this expression is so obvious, it's easy to miss — like trying to see your nose. In any case, here it is:

$$\sqrt{a} \cdot \sqrt{a} = a$$

In other words, **a square root times itself equals the very number it's the square root of.** Since this idea comes from the defintion of a square root (**p. 116**), it's true for any number or term. You could even say something ridiculous, like:

$$\sqrt{\textbf{elephant}} \cdot \sqrt{\textbf{elephant}} = \textbf{elephant},$$

and it would be true! To your right are more examples of this concept.

Examples

$$\sqrt{9} \cdot \sqrt{9} = 9$$

$$\sqrt{7x} \cdot \sqrt{7x} = 7x$$

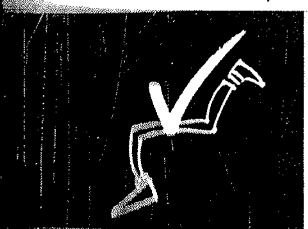
$$\sqrt{ax^2} \cdot \sqrt{ax^2} = ax^2$$

$$\sqrt{7xyz^2} \cdot \sqrt{7xyz^2} = 7xyz^2$$

Silly example

$$\sqrt{\textbf{chocolate}} \cdot \sqrt{\textbf{chocolate}} = \textbf{chocolate}$$

Note: in Algebra I, the square root of a negative number, e.g. $\sqrt{-5}$, is considered "undefined." That means it gives you no real answer.

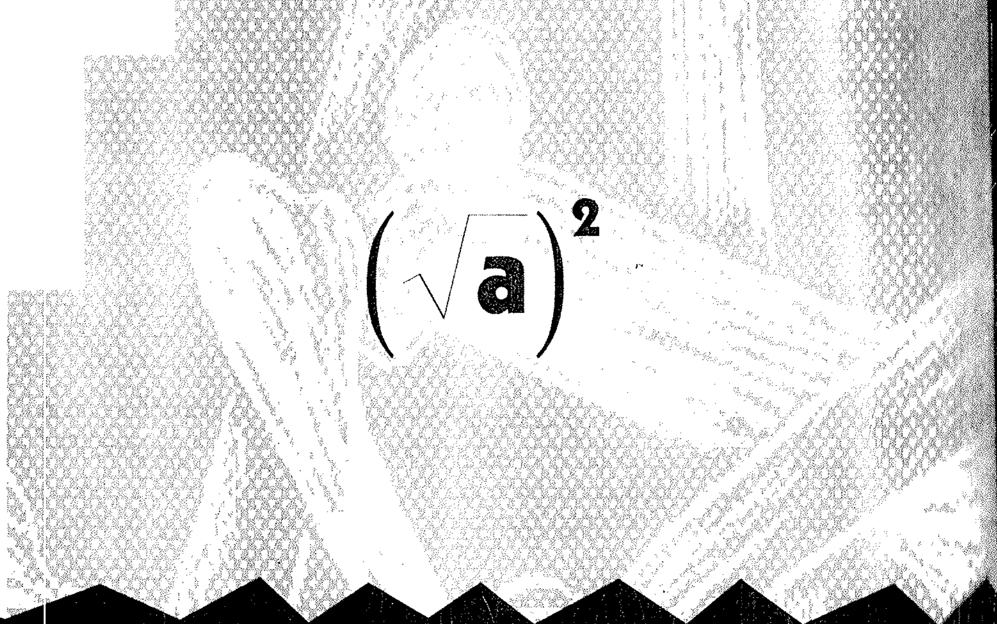


Now try these:

a) $\sqrt{y} \cdot \sqrt{y}$
b) $\sqrt{7} \cdot \sqrt{7}$
c) $\sqrt{200ab} \cdot \sqrt{200ab}$
d) $\sqrt{\text{pizza}} \cdot \sqrt{\text{pizza}}$
e) $\sqrt{45x^2y} \cdot \sqrt{45x^2y}$

Answers:
a) y
b) 7
c) 200ab
d) pizza
e) $45x^2y$

Q: O.K. Now I understand the value of terms like: $\sqrt{a} \cdot \sqrt{a}$ But what would be the value of a weird-looking term like this:



$$\left(\sqrt{a}\right)^2$$

A: Don't let the weird look of this term toss you out of the saddle. Remember from the **Exponents** section (**p. 86**) that a term to the second power means that the term is just multiplying itself. In other words: $\left(\sqrt{a}\right)^2 = \sqrt{a} \cdot \sqrt{a}$

On the previous page you saw that: $\left(\sqrt{a}\right) \cdot \left(\sqrt{a}\right) = a$

And the transitive property (**p. 11**) says that when all terms are equal to each other, the term you start with equals the term you end up with. So the point is this:

$$\left(\sqrt{a}\right)^2 = a$$

Examples

$\left(\sqrt{9}\right)^2 = 9$

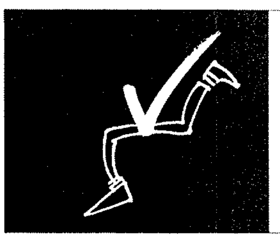
$\left(\sqrt{7x}\right)^2 = 7x$

$\left(\sqrt{ax^2}\right)^2 = ax^2$

$\left(\sqrt{17pqr}\right)^2 = 17pqr$

Silly example

$\left(\sqrt{\text{hotdog}}\right)^2 = \text{hotdog}$



Try these:

a) $\left(\sqrt{j}\right)^2$

b) $\left(\sqrt{pq}\right)^2$

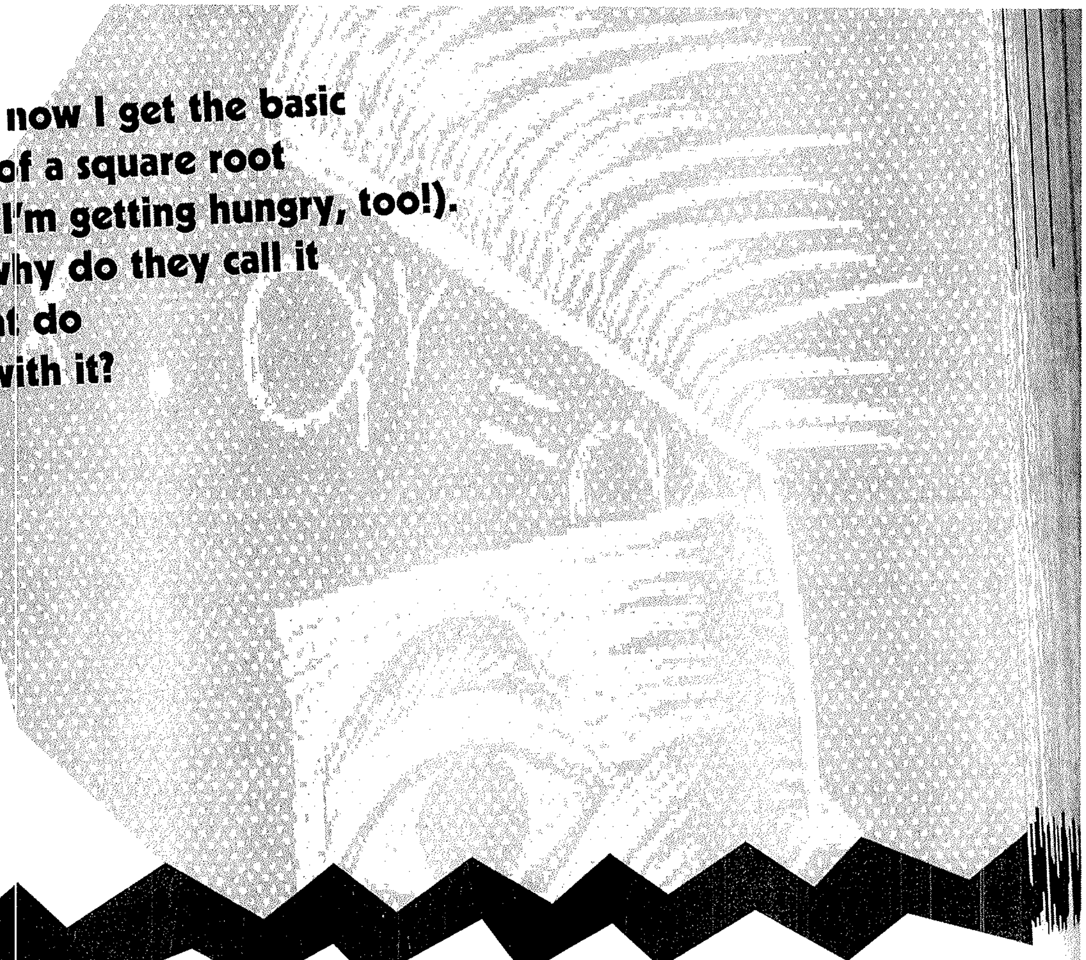
c) $\left(\sqrt{300}\right)^2$

d) $\left(\sqrt{\text{hamburger}}\right)^2$

e) $\left(\sqrt{11r^2}\right)^2$

Answers:
a) j
b) pq
c) 300
d) hamburger
e) $11r^2$

Q: **O.K., now I get the basic idea of a square root (and I'm getting hungry, too!). But why do they call it a "square" root? What do squares have to do with it?**

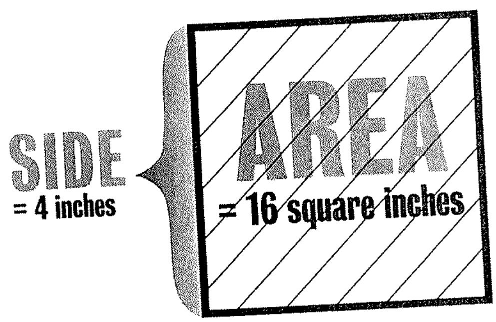


A: **It's called a "square" root because you can understand its length by thinking about a square.** Here's how: pick a number, any number. Then find a square whose area is equal to that number. Next measure the length of a side of this square. The length of that side, it turns out, is the square root of the number you started with. Presto, nothing up the sleeve. And ... now that you may be more confused than ever, here's an example to help you out.

Example

You know from geometry that the area of a square is just the length of one of its sides times itself. For example, if a side of a square is **4 inches**, the area of this square would be:

(**4 inches**) · (**4 inches**) or **16 square inches**.

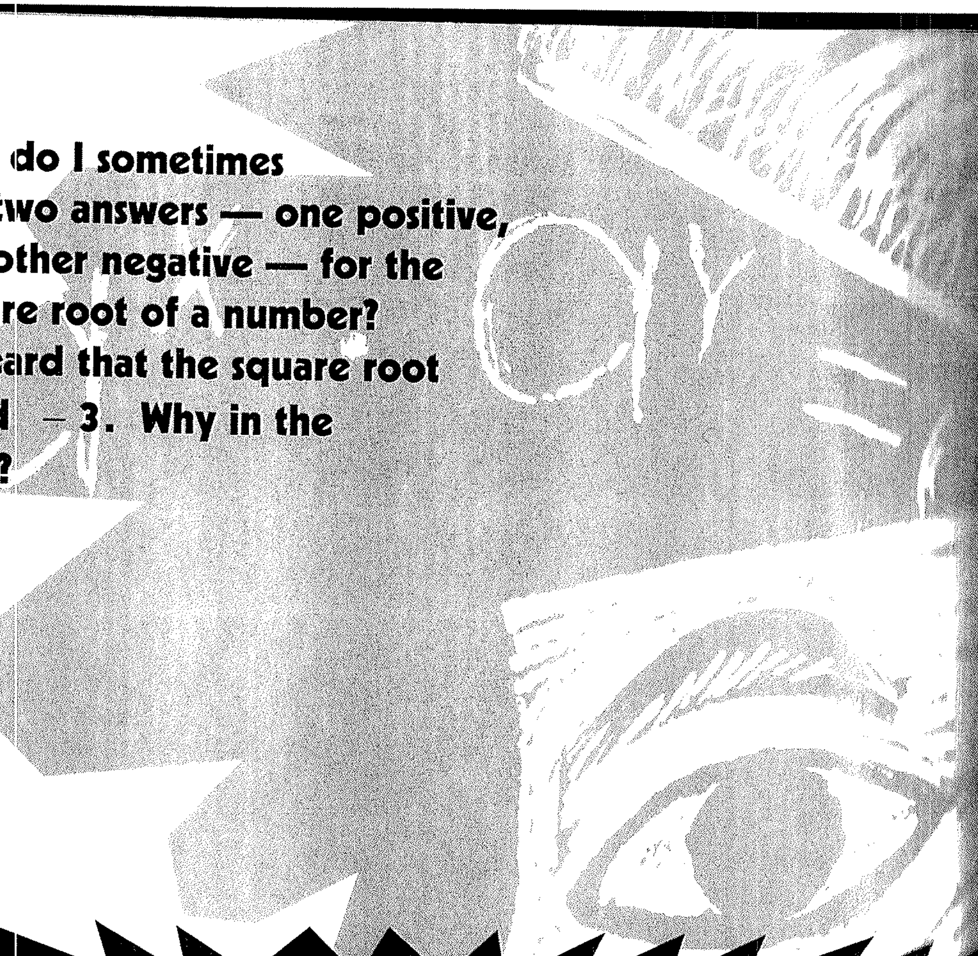


But if you think about it, **4** is the square root of **16**. In other words, the length of the side is the square root of the area of the square.

Length of side	=	**square root of area of the square**
4 inches	=	$\sqrt{16}$ **square inches**

Q: Why do I sometimes see two answers — one positive, the other negative — for the square root of a number?

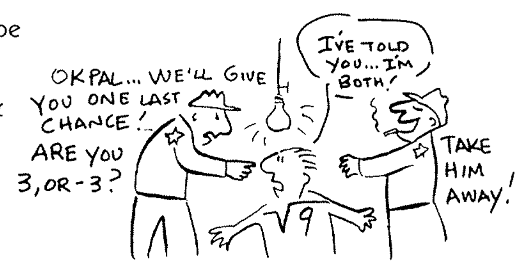
For example, I've heard that the square root of 9 is both $+3$ and -3. Why in the world would this be?

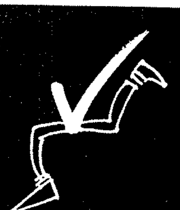


A: What you've heard is true. The square root of a positive number does have two answers — one positive, the other negative.

> In Algebra I, teachers rarely ask you to write both answers. Teachers usually want only the positive answer. Check with your instructor to find out her/his preference.

But why does the square root of nine equal negative three as well as positive three? Well, just think about it. The multiplication rule (**p. 49**) tells you that $(-3)\cdot(-3) = +9$. Therefore -3 must be a square root of 9 because it fits the definition of a square root (**p. 116**). To show both the positive and negative value of $\sqrt{9}$, you write: $\sqrt{9} = \pm 3$. This means that the square root of 9 is both $+3$ and -3.





Give both values for these square roots:

a) $\sqrt{4}$
b) $\sqrt{25}$
c) $\sqrt{49}$
d) $\sqrt{81}$
e) $\sqrt{100}$

Answers:
a) ± 2
b) ± 5
c) ± 7
d) ± 9
e) ± 10

Q: Now I thoroughly understand what a square root is. But what is a perfect square?

A: A perfect square is what you get when you square a number or a term; that is, when you multiply it by itself.

Examples

9 is the perfect square of **3** because $3 \cdot 3 = 9$

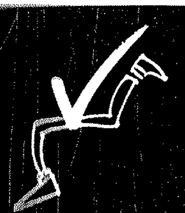
a^2 is the perfect square of **a** because $a \cdot a = a^2$

$9a^2$ is the perfect square of **3a** because

$$(3a) \cdot (3a) = 9a^2$$

Some perfect squares to memorize

#	→	perfect square	#	→	perfect square
1	→	1	9	→	81
2	→	4	10	→	100
3	→	9	11	→	121
4	→	16	12	→	144
5	→	25	13	→	169
6	→	36	14	→	196
7	→	49	15	→	225
8	→	64	16	→	256

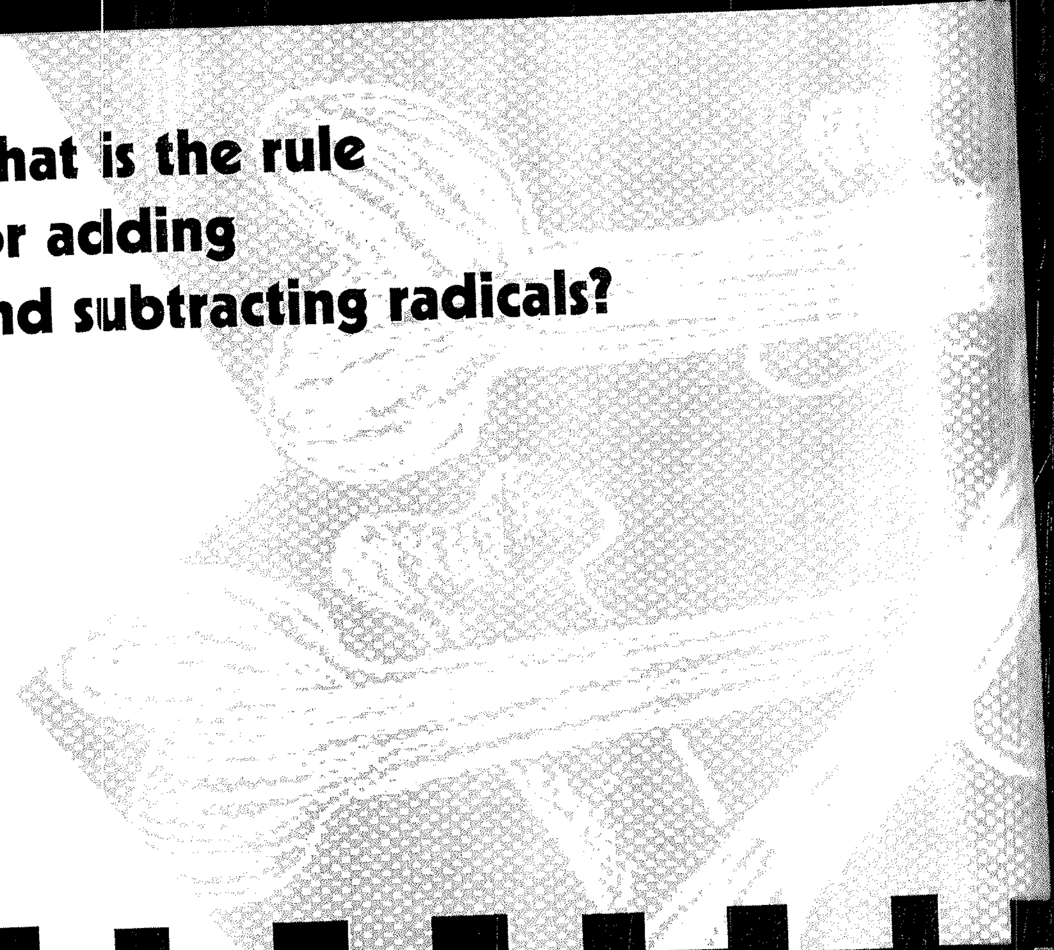


Find the perfect squares of these terms:

a) 17
b) 20
c) y
d) 9t
e) 100

Answers:
a) 289
b) 400
c) y^2
d) $81t^2$
e) 10, 000

Q: What is the rule for adding and subtracting radicals?



A: The rule for adding and subtracting radicals is simple. As long as what's under the radical sign is exactly the same, you can treat radicals as if they were "like terms" (**p. 66**).

In other words, terms like $2\sqrt{7}$ and $5\sqrt{7}$ are like terms because they both show a **7** under the radical. Since such terms are like terms, you can combine them using the same-, mixed- and neighbor-sign rules (**pp. 36-45**).

Combining radicals using the same-, mixed- and neighbor-sign rules

Same-sign rule

$$-4\sqrt{5} - 7\sqrt{5}$$
$$= -(4\sqrt{5} + 7\sqrt{5})$$
$$= -11\sqrt{5}$$

Mixed-sign rule

$$+4\sqrt{5} - 7\sqrt{5}$$
$$= -(7\sqrt{5} - 4\sqrt{5})$$
$$= -3\sqrt{5}$$

Neighbor-sign rule

$$4\sqrt{5} - -7\sqrt{5}$$
$$= 4\sqrt{5} + 7\sqrt{5}$$
$$= 11\sqrt{5}$$

Now try these:

a) $+5\sqrt{7} + 3\sqrt{7}$
b) $-2\sqrt{9} + 13\sqrt{9}$
c) $8\sqrt{5} + -6\sqrt{5}$
d) $+17\sqrt{2} - 21\sqrt{2}$
e) $-6\sqrt{11} - 3\sqrt{11}$
f) $2\sqrt{14} - -3\sqrt{14}$

Answers:
a) $+8\sqrt{7}$
b) $+11\sqrt{9}$
c) $+2\sqrt{5}$
d) $-4\sqrt{2}$
e) $-9\sqrt{11}$
f) $5\sqrt{14}$

Q: Now I see that I can combine radicals when they have the same numbers beneath them. But can I also combine radicals containing the same variables or groups of variables?

A: Yes, you may. Again, the rule is that as long as what's under the radical sign is the same, you can combine radicals using the same-, mixed- and neighbor-sign rules.

Examples showing how radicals with variables can be combined

Same-sign rule

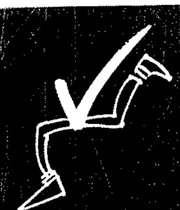
$$+\,3\sqrt{m} + 8\sqrt{m}$$
$$= +\left(3\sqrt{m} + 8\sqrt{m}\right)$$
$$= +\,11\sqrt{m}$$

Mixed-sign rule

$$-\,4\sqrt{x} + 11\sqrt{x}$$
$$= +\left(11\sqrt{x} - 4\sqrt{x}\right)$$
$$= +\,7\sqrt{x}$$

Neighbor-sign rule

$$9\sqrt{vw} + \left(-\,3\sqrt{vw}\right)$$
$$= 9\sqrt{vw} - 3\sqrt{vw}$$
$$= 6\sqrt{vw}$$



Now try these:

a) $-\,4\sqrt{u} - 5\sqrt{u}$

b) $12\sqrt{w} + \left(+\,2\sqrt{w}\right)$

c) $-\,3\sqrt{n} + 13\sqrt{n}$

d) $15\sqrt{r} - \left(-\,2\sqrt{r}\right)$

e) $+\sqrt{x} + \sqrt{x}$

f) $+\sqrt{c} - 6\sqrt{c}$

Answers:

a) $-\,9\sqrt{u}$

b) $14\sqrt{w}$

c) $+\,10\sqrt{n}$

d) $17\sqrt{r}$

e) $2\sqrt{x}$

f) $-\,5\sqrt{c}$