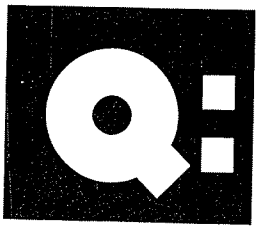
What is cancelling? And why should I learn how to cancel?

Ever felt the need to tidy up your room? If so, you should be able to understand the need for cancelling, for cancelling is the art of tidying up mathematical expressions.

To grasp the idea, imagine that one day, as you enter your bedroom, it's so filthy that you're knee-deep in trash. Despite yourself, you shout: "I can't take it any more!" So you grab a wastebasket and furiously start tossing out junk. Eventually you find that beneath all the trash you still have a pretty nice room.

In math, you don't have sloppy rooms to tidy, but you do have messy mathematical expressions. And instead of using a garbage pail, you need only grab your pencil, for with it you can cross out and toss out mathematical garbage. After you get rid of mathematical clutter, you'll be left with a simple and beautiful mathematical expression.

Like cleaning up your room, cancelling may be less than fun to do, but once you've done it, you feel so much better!



Well, cancelling sounds fine, but I'd like to see an example of the process so I can better understand it.

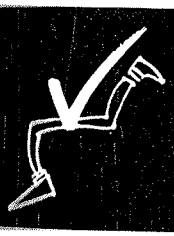
Sure. Cancelling involves finding factors that stand over themselves in fractions, then crossing them out. After cancelling factors, you reduce their value to **1/1** or just plain **1**. Below are two examples.

Examples of the process of cancelling

You see the term: **3a/3**. "Hmmm," you say, "I bet I could tidy that up." Knowing that **3/3** = **1/1**, you strike out the **3**'s like this: ~~**3**~~**a**/~~**3**~~, and rewrite the fraction as: **1a/1**. But you can do even better than that, for you know the numerator, **1a**, equals **a**. So you rewrite **1a/1** as: **a/1**. Finally, you know that **a/1** simply means **a**. Congratulations! You just reduced the messy fraction **3a/3** to the tidier term: **a**

Use a similar approach to reduce a fraction with a variable in the denominator, a fraction like: **3/3a**. First cancel the **3**'s, to get: ~~**3**~~/~~**3**~~**a** = **1/1a**. Again, since **1a** = **a**, you say: **1/1a** = **1/a** That's it. You're done! **3/3a** reduces to **1/a**.

Of course, once you become a cancelling whiz, you won't need to go through these steps in such slow, agonizing detail. With one swift stroke of the pencil, you'll reduce **3a/3** to **a** or **3/3a** to **1/a**.



Cancel and reduce these fractions:

a) 5x/5
b) 5/5x
c) 7tu/7
d) 7/7tu

Answers:
a) x
b) 1/x
c) tu
d) 1/tu

Q: What is reducing? And how does reducing differ from cancelling?

A: Reducing and cancelling are like two sides of the same coin in the sense that they're two aspects of the same process.

Still it helps to distinguish the terms since they have slightly different meanings.

Cancelling

Strictly speaking, **cancelling** means **spotting identical factors on opposite sides of the fraction bar, then grabbing your pencil and crossing them out**. Since getting rid of junk feels good, cancelling gives you that same good feeling of getting rid of stuff you no longer need.

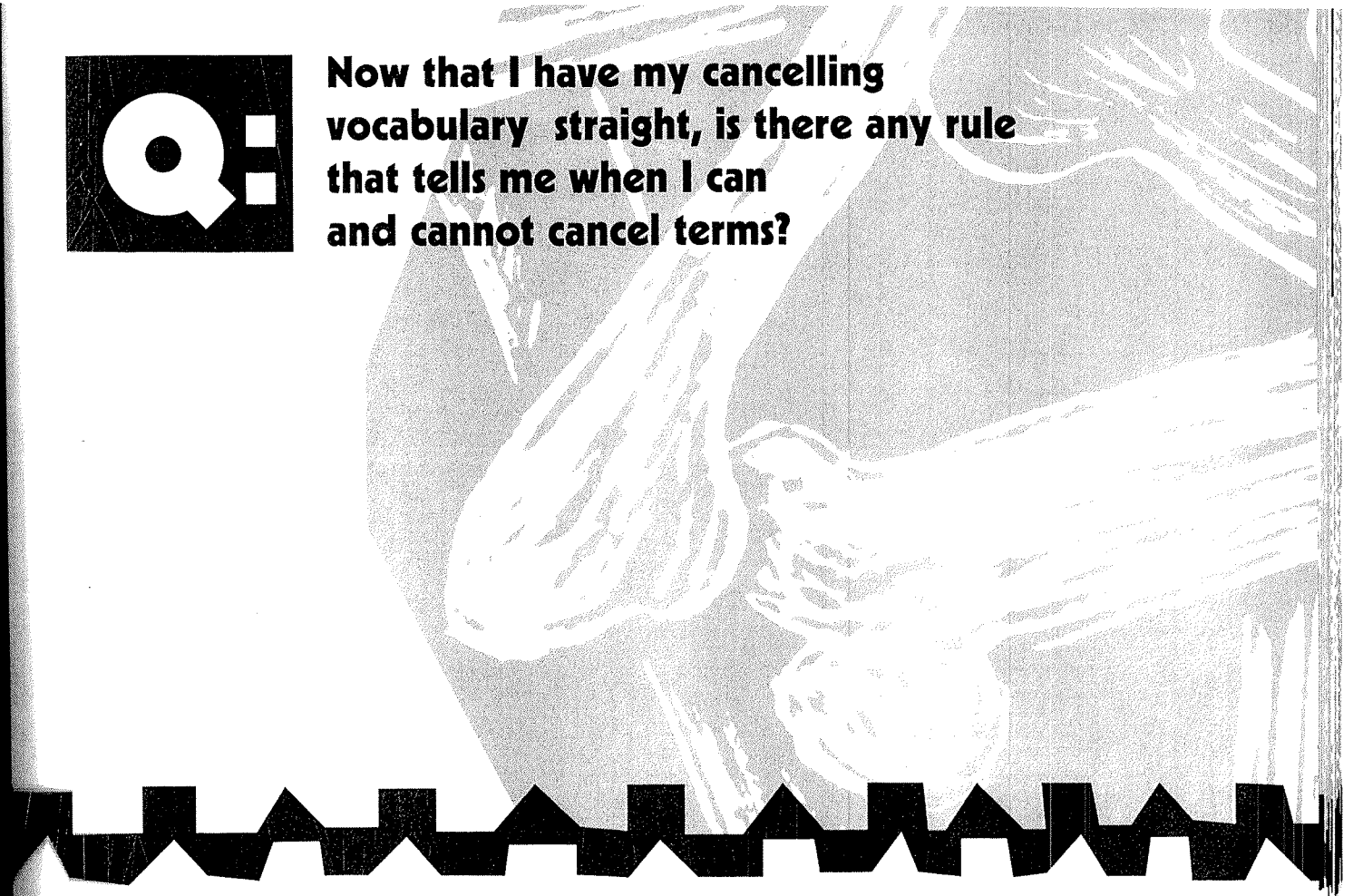
Reducing

Reducing means **shrinking** something, boiling it down to a more manageable size. So what do you reduce? You **reduce** the terms you've cancelled by changing their value to the more manageable form of **1/1** or **1**. By doing that, you also **reduce** the entire term to a more manageable form.

Illustration of the use of these terms

Look at the first example on the last page, **3a/3**. You **cancel** the **3**'s when you spot them and cross them out.

Then you **reduce** **3/3** to the more manageable form of **1/1**. By doing all of this, you **reduce** the entire term, **3a/3**, to the simpler term: **a**



Q: Now that I have my cancelling vocabulary straight, is there any rule that tells me when I can and cannot cancel terms?

A: In fact there is. It's called the **cancelling rule**, and here's what it says: **You may cancel only when you have identical factors on opposite sides of the fraction bar.**

If you don't have this situation, you can **never** cancel, no matter how tempted you are to whip out your pencil and start wiping out terms. Knowing this rule will save you headaches, so please do yourself a favor: **memorize** it.

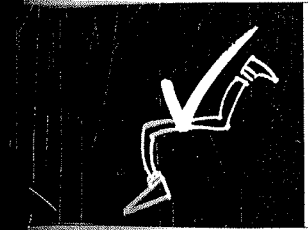
How the cancelling rule works in real life

In the previous example of **3a/3**, you may cancel the **3**'s only because they are **identical factors on opposite sides of the fraction bar**.

In other words —

Fraction is: **3a/3**
Factors of numerator are: **1**, **3** and **a**
Factors of denominator are: **1** and **3**

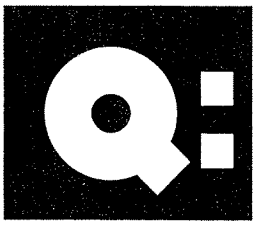
The factors that are **identical** in both the numerator and denominator are these: **1** and **3**. Cancelling **1**'s never does anything to reduce a fraction's value (try it and see), so don't bother with the **1**'s. But you can and should cancel the two **3**'s.



Tell what term(s) you can cancel in the following fractions:

a) $\frac{xy}{x}$ b) $\frac{4bc}{4b}$ c) $\frac{zy(p+q)}{z(v+w)}$ d) $\frac{xy(p+q)}{vw(p+q)}$

Answers:
a) x c) z
b) 4 and (p + q) (b + d)



When terms cancel, they just disappear, don't they? And since that's true, I never have to worry about them again, right?

No, not so fast!

Cancelled terms never "disappear"; they reduce to **1/1** or **1**. This matters because cancelled terms will sometimes pop their heads up and appear in your answer. So ... no, you can't just forget about cancelled terms. Here's the rule:

Cancelled terms **fade away** when they're linked to other terms by **multiplication or division**. But they **appear** — and affect your answer — when they're linked by **addition or subtraction**. To the right are examples.

cancelled terms fade away

$$\frac{ac}{a} = \frac{\cancel{a}c}{\cancel{a}} = c$$

The **a**'s, which cancel, **fade away** from your answer. That's because they're linked to the other term, **c**, by **multiplication and division.**

cancelled terms appear in answer

$$\frac{a}{a} + \frac{c}{a} = \frac{\cancel{a}}{\cancel{a}} + \frac{c}{a} = 1 + \frac{c}{a}$$

But here the cancelled **a**'s turn into **1** and **appear** in your answer. That's because they're linked to another term by **addition**.

Cancel and give the simplified form:

a) $\frac{xy}{x}$ c) $8 - \frac{a}{a}$

b) $\frac{x}{y} + \frac{x}{x}$ d) $\frac{4cd}{cd}$

Answers: a) y c) 7 b) $1 + \frac{x}{y}$ d) 4

Q: How would I reduce fractions in which I can factor either the numerator or the denominator, fractions like these:

$$\frac{a + ac}{a} \quad \text{and} \quad \frac{a}{a + ac}$$

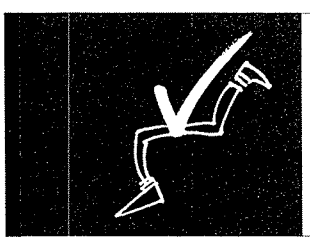
A: You factor whatever you can factor. Just follow the **F.C.R.** steps, as shown to the right.

Reduce the fractions:

A) $\dfrac{a + ac}{a}$,

B) $\dfrac{a}{a + ac}$

Steps	**Example A**	**Example B**
1st) Factor.	$\dfrac{a + ac}{a} = \dfrac{a(1 + c)}{a}$	$\dfrac{a}{a + ac} = \dfrac{a}{a(1 + c)}$
2nd) Cancel.	$= \dfrac{\cancel{a}(1 + c)}{\cancel{a}}$	$= \dfrac{\cancel{a}}{\cancel{a}(1 + c)}$
3rd) Reduce.	$= 1 + c$	$= \dfrac{1}{1 + c}$



Try simplifying these fractions using the F.C.R. steps:

a) $\dfrac{x + xy}{x}$ c) $\dfrac{mn + mp}{m}$

b) $\dfrac{d}{d + de}$ d) $\dfrac{t}{tu - tv}$

Answers:

a) $1 + y$ c) $d + u$

b) $\dfrac{a + 1}{1}$ (q

$\dfrac{n - n}{1}$ (p

Q: How would I reduce a fraction in which I can factor both the numerator and the denominator, a fraction like this:

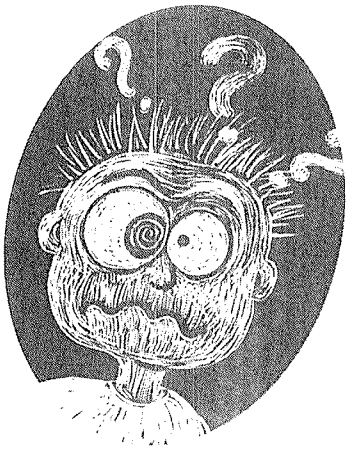
$$\frac{acm + acn}{acu + acv}$$

A: You use the same **F.C.R.** steps, only here you need to factor both the numerator and the denominator. Follow the steps to the right.

Reduce the fraction:

$$\frac{acm + acn}{acu + acv}$$

Steps	**Example**
1st) Factor.	$\frac{acm + acn}{acu + acv} = \frac{ac(m + n)}{ac(u + v)}$
2nd) Cancel.	$= \frac{\cancel{ac}(m + n)}{\cancel{ac}(u + v)}$
3rd) Reduce.	$= \frac{m + n}{u + v}$



Now try reducing these fractions:

a) $\frac{xy + xz}{xp + xq}$ c) $\frac{wxy - wyz}{awy - wyb}$

b) $\frac{ced + cev}{cew - cez}$ d) $\frac{qx + qw - qr}{qv + qy}$

Answers:

a) $\frac{y + z}{b + d}$ c) $\frac{x - z}{a - q}$

b) $\frac{d + w}{z - w}$ d) $\frac{r - w + x}{v + v}$

Q: How would I reduce a fraction that has terms raised to exponents, a fraction like this:

$$\frac{acm + acn}{a^2cu + a^2cv}$$

A: You still follow the F.C.R. steps. Follow the example shown to the right.

Reduce the fraction:

$$\frac{acm + acn}{a^2cu + a^2cv}$$

Steps	**Example**
1st) Factor.	$\frac{acm + acn}{a^2cu + a^2cv} = \frac{ac(m + n)}{a^2c(u + v)}$
2nd) Cancel.	$= \frac{\cancel{ac}(m + n)}{a^{\cancel{2}}\cancel{c}(u + v)}$
3rd) Reduce.	$= \frac{m + n}{a(u + v)}$

To understand the cancelling step here, think of $\frac{a}{a^2}$ as $\frac{a}{a \cdot a}$. When you cancel terms here, the single **a** in the numerator cancels with only one of the **a**'s in the denominator, like this: $\frac{\cancel{a}}{a \cdot \cancel{a}}$. This tells you why the final answer still has an **a** in the denominator.

Try reducing these fractions:

a) $\frac{xy + xz}{x^2w + x^2a}$

b) $\frac{ab + ac}{a^2d - a^2w}$

c) $\frac{m^2n^3p + m^2n^2q}{m^2n^2}$

d) $\frac{x^3y^2 - x^2y^4}{x^2y^2}$

Answers:

a) $\frac{yx + wx}{z + y}$

b) $b + du$

c) $\frac{pe - we}{c + q}$

d) $y^2 - x$