Q: I understand the three previous examples. But sometimes I get confused about what I can and cannot cancel. For example, in the problem:

$$\frac{a}{a+c},$$

it seems like I should be able to cancel the a in the numerator with the a in the denominator. But my teacher says I can't. Why can't I cancel those two a's?

A: Excellent question. In $\frac{a}{a+c}$, **a** is a factor of the numerator, but for you to cancel the two **a**'s, **a** also would need to be a factor of the denominator (**p. 171**). So the question is: why <u>isn't</u> **a** a factor of the denominator? To answer this, let's remember what it means to be a factor.

Refresher course: what's a factor?

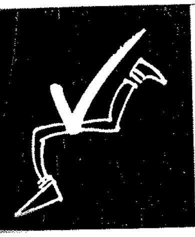
The definition of a factor (**pp. 142-145**) says: **for a term to be a factor of some expression, you need to be able to multiply that term by something to get the expression.**

For example, **3** is a factor of **21** because $3 \cdot \text{something} = 21$, namely: $3 \cdot 7 = 21$. And **6a** is a factor of $6a^2 + 12a$ because $6a \cdot \text{something} = 6a^2 + 12a$, namely:

$$6a \cdot (a+2) = 6a^2 + 12a$$

Going back to the question ...

Now that that's clear, look again at the denominator of $\frac{a}{a+c}$. For **a** to be a factor of the denominator, **a + c**, you'd need to be able to multiply **a** by something to get **a + c**. Can you? You can search, but the answer is "no." That means that **a** is not a factor of **a + c**. And that means that the two **a**'s in the fraction don't cancel, for you can cancel only factors (**p. 171**).



Tell whether or not the variable a is a factor of the expression:

a) $5a$
b) $5 + a$
c) $a(b + c)$
d) $a + b + c$
e) $x(a - b)$
f) $a(a - b)$
g) $ab(a - b)$
h) $bc(b - a)$

Answers:
a) is a factor
b) isn't a factor
c) is a factor
d) isn't a factor
e) isn't a factor
f) is a factor
g) is a factor
h) isn't a factor

Q: I'm beginning to see that when a term is linked to other terms only by addition or subtraction, it's not a factor. But when it's linked by multiplication, it is a factor. What I'm wondering about now is something a little different.

In a fraction like: $\frac{a}{a + c}$, are there any factors at all for the denominator?

A: Another good question, for it brings up a crucial point. Even though neither **a** nor **c** is a factor of the denominator, still the whole expression: $(a + c)$ is a factor of the denominator.

Why? Well, can't you rewrite the denominator as: $1 \cdot (a + c)$?

Since you can, you can view the whole quantity: $(a + c)$ as a factor of the denominator.

Burn this into your mind: the **whole numerator** and the **whole denominator** are always considered **factors of themselves**. This idea is critical, for as you'll soon see, it allows you to do a special kind of cancelling.

> When you're talking about some quantity as a factor, teachers usually want you to stick that quantity inside parentheses. The parentheses show that the factor is the whole quantity, not the individual terms that make it up. For example, since $a + c$ is a factor of the denominator in $\frac{1}{a + c}$, you'd write it as: $(a + c)$, and you'd read it as "the quantity $a + c$."

Name the factor(s) of the numerator and of the denominator for these fractions:

a) $\frac{a(b - c)}{b - c}$ **b)** $\frac{x - y + z}{x(y + z)}$

Answers:
a) factors of numerator: a and $(b - c)$
factor of denominator: $(b - c)$
b) factor of numerator: $(x - y + z)$
factors of denominator: x and $(y + z)$

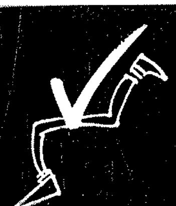
Q: Well then ... if an entire numerator or an entire denominator may be viewed as a factor, does that mean I can cancel an entire numerator or denominator?

In other words, would I be able to cancel and reduce fractions like these:

$$\frac{a+c}{a+c},\quad \frac{3(a+c)}{a+c}\quad \text{and}\quad \frac{a+c}{3(a+c)}$$

A: Yes. Here's how. **Simplify:** **A)** $\frac{a+c}{a+c}$, **B)** $\frac{3(a+c)}{a+c}$, **C)** $\frac{a+c}{3(a+c)}$

Steps	Example A	Example B	Example C
1st) Spot numerators or denominators that are factors of themselves. Put them in parentheses to show that they're factors of themselves.	$\frac{a+c}{a+c} = \frac{(a+c)}{(a+c)}$	$\frac{3(a+c)}{a+c} = \frac{3(a+c)}{(a+c)}$	$\frac{a+c}{3(a+c)} = \frac{(a+c)}{3(a+c)}$
2nd) Cancel.	$= \frac{\cancel{(a+c)}}{\cancel{(a+c)}}$	$= \frac{3\cancel{(a+c)}}{\cancel{(a+c)}}$	$= \frac{\cancel{(a+c)}}{3\cancel{(a+c)}}$
3rd) Reduce.	$= 1$	$= 3$	$= \frac{1}{3}$



Try reducing these fractions:

a) $\frac{x+y}{x+y}$ c) $\frac{7(p+q-z)}{p+q-z}$

b) $\frac{x-y}{4(x-y)}$ d) $\frac{6(m-n)}{8(m-n)}$

Answers: a) 1 b) $\frac{1}{4}$ c) 7 d) $\frac{3}{4}$

Q: **I understand that I can't reduce a fraction like:** $\dfrac{a}{a + c}$

But what about the flip of that, a fraction like: $\dfrac{a + c}{a}$

Can I reduce this kind of fraction?

A: After a moment of study, you're probably ready to say this can't be reduced because there are no factors common to the numerator and denominator. And in that sense, you'd be absolutely right.

However you're now plunging into a "grey area" of algebra.

While many teachers would accept $\dfrac{a + c}{a}$ as an answer in final form, others would ask you to simplify it by using a trick.

To understand this trick, you first need to learn a rule called the **split-the-numerator rule**. The next several pages will teach you how this rule works.

Q: What does the split-the-numerator rule say?

$$\frac{a + b - c + d}{x} = ?$$

A: The split-the-numerator rule says this:

$$\frac{a + b - c + d}{x} = \frac{a}{x} + \frac{b}{x} - \frac{c}{x} + \frac{d}{x}$$

Or, in English: When a fraction's numerator has two or more terms linked by "+" or "−" signs, you can rewrite the fraction by:

a) splitting the numerator into a series of numerators, each of which stands over the denominator, and

b) hitching the fractions together by placing "+" or "−" signs at the appropriate spots between them.

Examples of the split-the-numerator rule

$$\frac{m + p - q}{r} = \frac{m}{r} + \frac{p}{r} - \frac{q}{r}$$

$$\frac{x^2 - 2y + z}{3q} = \frac{x^2}{3q} - \frac{2y}{3q} + \frac{z}{3q}$$

Silly example

$$\frac{ape - cow}{cat} = \frac{ape}{cat} - \frac{cow}{cat}$$

Rewrite using the split-the-numerator rule:

a) $\frac{c + e}{v}$ c) $\frac{3 - x^3 + y^2}{b^3}$

b) $\frac{q^2 - 9q}{11}$ d) $\frac{7p + g + m}{4h}$

Answers:

a) $\frac{c}{v} + \frac{e}{v}$ b) $\frac{q^2}{11} - \frac{9q}{11}$ c) $\frac{3}{b^3} - \frac{x^3}{b^3} + \frac{y^2}{b^3}$ d) $\frac{7p}{4h} + \frac{g}{4h} + \frac{m}{4h}$

Q: Can I also use the split-the-numerator rule when I have a complex denominator? In other words, can I use the rule for fractions like these:

$$\frac{2a - b}{a + b} \quad \text{and} \quad \frac{2a - b}{x(a + b)}$$

A: Yes, you can use the split-the-numerator rule no matter how complex your denominator might be. What you need to decide, though, is whether using the rule makes your answer look more simple or more complicated. Teachers differ on this point. Check with your teacher to learn his/her preference.

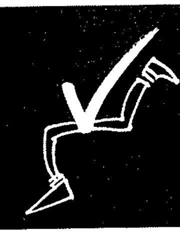
Examples of the split-the-numerator rule

$$\frac{2a - b}{a + b} = \frac{2a}{a + b} - \frac{b}{a + b}$$

$$\frac{2a + b}{x(a + b)} = \frac{2a}{x(a + b)} + \frac{b}{x(a + b)}$$

Silly Example

$$\frac{\text{ape} - \text{cow}}{\text{cat} + \text{dog}} = \frac{\text{ape}}{\text{cat} + \text{dog}} - \frac{\text{cow}}{\text{cat} + \text{dog}}$$



Split the numerators of these fractions:

a) $\frac{a + b}{c - d}$ c) $\frac{x^2 - 9}{x + y}$

b) $\frac{a - b}{c - d}$ d) $\frac{x^2 + 9}{x + y}$

Answers:

a) $\frac{a}{c - d} + \frac{b}{c - d}$ c) $\frac{x^2}{x + y} - \frac{9}{x + y}$

b) $\frac{a}{c - d} - \frac{b}{c - d}$ d) $\frac{x^2}{x + y} + \frac{9}{x + y}$

Q: Now I understand the split-the-numerator rule, but how does it help me simplify the fraction I saw a few pages back, this fraction:

$$\frac{a + c}{a}$$

A: If you use the split-the-numerator rule on this fraction, the **a**'s will cancel nicely. Follow the steps to the right.

Simplify:

$$\frac{a + c}{a}$$

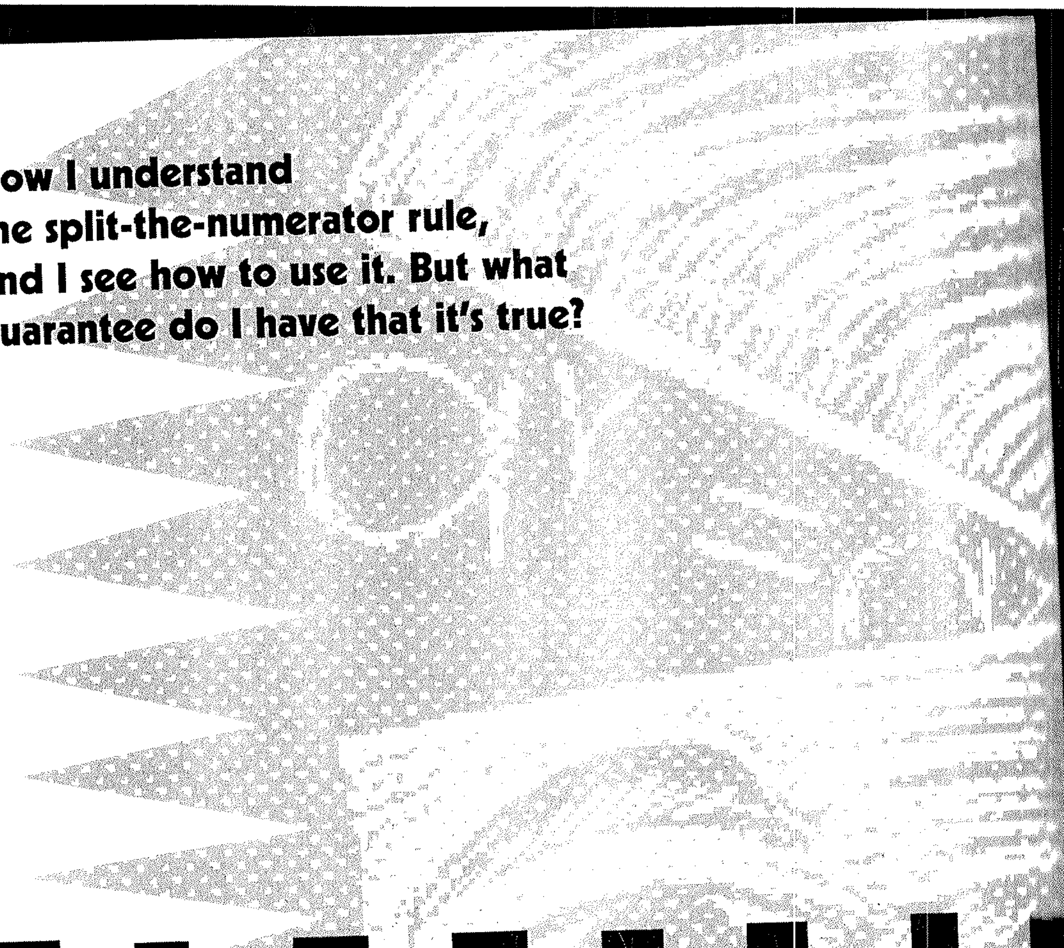
Steps	Example
	$\frac{a + c}{a}$
1st) Use the split-the-numerator rule (**p. 181**).	$= \frac{a}{a} + \frac{c}{a}$
2nd) **C**ancel.	$= \frac{\not{a}}{\not{a}} + \frac{c}{a}$
3rd) **R**educe.	$= 1 + \frac{c}{a}$

Try simplifying these fractions with the split-the-numerator rule:

a) $\frac{x + y}{x}$ c) $\frac{a^2 + b}{a^2}$

b) $\frac{y - x}{x}$ d) $\frac{a^2 + b}{a}$

Answers: a) $1 + \frac{y}{x}$ b) $\frac{y}{x} - 1$ c) $1 + \frac{b}{a^2}$ d) $a + \frac{b}{a}$



Q: Now I understand the split-the-numerator rule, and I see how to use it. But what guarantee do I have that it's true?

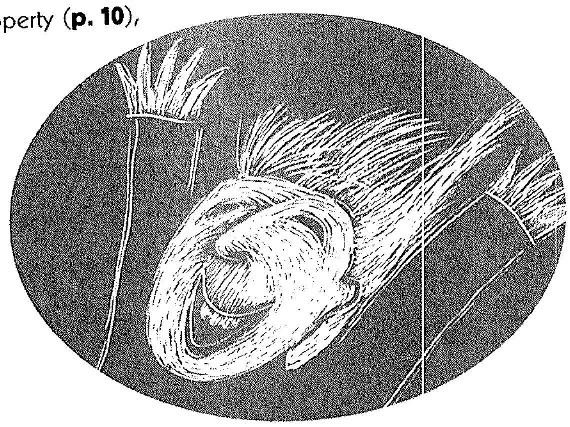
A: Good question. And fortunately, the answer is quite simple, for it stems from a basic fraction rule you learned back in elementary school.

As you recall, when adding fractions with the same denominator, you just add the numerators and keep the denominator.

For example: $\frac{2}{7} + \frac{4}{7} = \frac{2+4}{7}$. Adding numerators, this answer will be: $\frac{6}{7}$

What's important is that thanks to the symmetric property (**p. 10**), you can flip the first statement around, to get:

$$\frac{2+4}{7} = \frac{2}{7} + \frac{4}{7}$$



But if you just ponder this last statement for a moment, you'll see that it uses the same idea as the split-the-numerator rule. In other words, the split-the-numerator rule is simply the algebraic version of a rule you already learned — way back in elementary school.

Q:

Cancel and reduce:

a) $\frac{7e}{7}$

b) $\frac{11}{11y}$

c) $\frac{pq}{p}$

d) $\frac{x}{y} + \frac{x}{x}$

e) $\frac{m}{mn - mr}$

f) $\frac{xy + xv}{xw + xz}$

g) $\frac{abc + abd}{ab^2e + ab^2f}$

Name the factors in the numerator and denominator:

h) $\frac{x + y}{a + b}$

j) $\frac{x(x + y)}{a + b}$

k) $\frac{x^2 - 5}{7(x^2 - 5)}$

Cancel and reduce:

l) $\frac{m + n}{m + n}$

m) $\frac{6(u - v)}{u - v}$

n) $\frac{p - q}{8(p - q)}$

Split the numerator and rewrite:

p) $\frac{x + y - z}{a}$

q) $\frac{x - y}{a - b}$

r) $\frac{a^2 - 3}{b^2 + 4}$

Split the numerator and reduce:

s) $\frac{p + q}{p}$

t) $\frac{q - p}{p}$

u) $\frac{b^2 + c}{b}$

A:

a) e

b) $\frac{1}{y}$

c) q

d) $\frac{x}{y} + 1$

e) $\frac{1}{n - r}$

f) $\frac{y + v}{w + z}$

g) $\frac{c + d}{be + bf}$

h) numerator: $(x + y)$
denominator: $(a + b)$

j) numerator: x and $(x + y)$
denominator: $(a + b)$

k) numerator: $(x^2 - 5)$
denominator: 7 and $(x^2 - 5)$

l) 1

m) 6

n) $\frac{1}{8}$

p) $\frac{x}{a} + \frac{y}{a} - \frac{z}{a}$

q) $\frac{x}{a - b} - \frac{y}{a - b}$

r) $\frac{a^2}{b^2 + 4} - \frac{3}{b^2 + 4}$

s) $1 + \frac{q}{p}$

t) $\frac{q}{p} - 1$

u) $b + \frac{c}{b}$