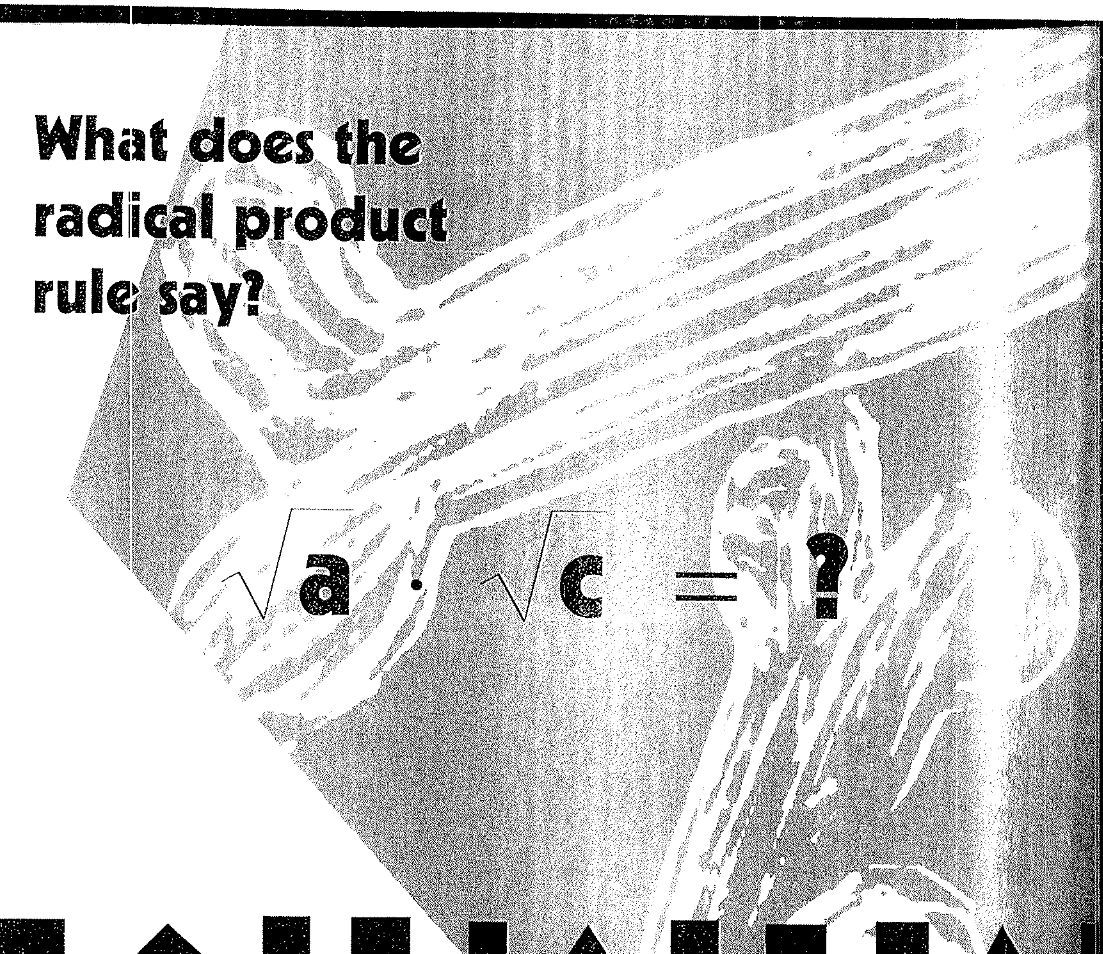
Q: What does the radical product rule say?



$$\sqrt{a} \cdot \sqrt{c} = ?$$

A: The radical product rule says you can simplify a product of radicals in this nice easy way:

$$\sqrt{a} \cdot \sqrt{c} = \sqrt{a \cdot c}$$

Here's how you can remember this rule: **When two or more radical terms are multiplying each other, you can simplify them by making one big, happy radical sign and multiplying all the terms underneath it.**

Examples of the radical product rule

$$\sqrt{m} \cdot \sqrt{v} = \sqrt{m \cdot v}$$

$$\sqrt{2} \cdot \sqrt{5} = \sqrt{2 \cdot 5} = \sqrt{10}$$

$$\sqrt{r} \cdot \sqrt{s} \cdot \sqrt{u} = \sqrt{r \cdot s \cdot u} = \sqrt{rsu}$$

$$\sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} = \sqrt{2 \cdot 3 \cdot 5} = \sqrt{30}$$

Note: This rule is very flexible. As the examples show, you can use it on any number of terms. And it makes no difference whether those terms are numbers, variables, or a combination of numbers and variables.

Combine using the radical product rule:

a) $\sqrt{2} \cdot \sqrt{3}$

b) $\sqrt{4} \cdot \sqrt{5} \cdot \sqrt{7}$

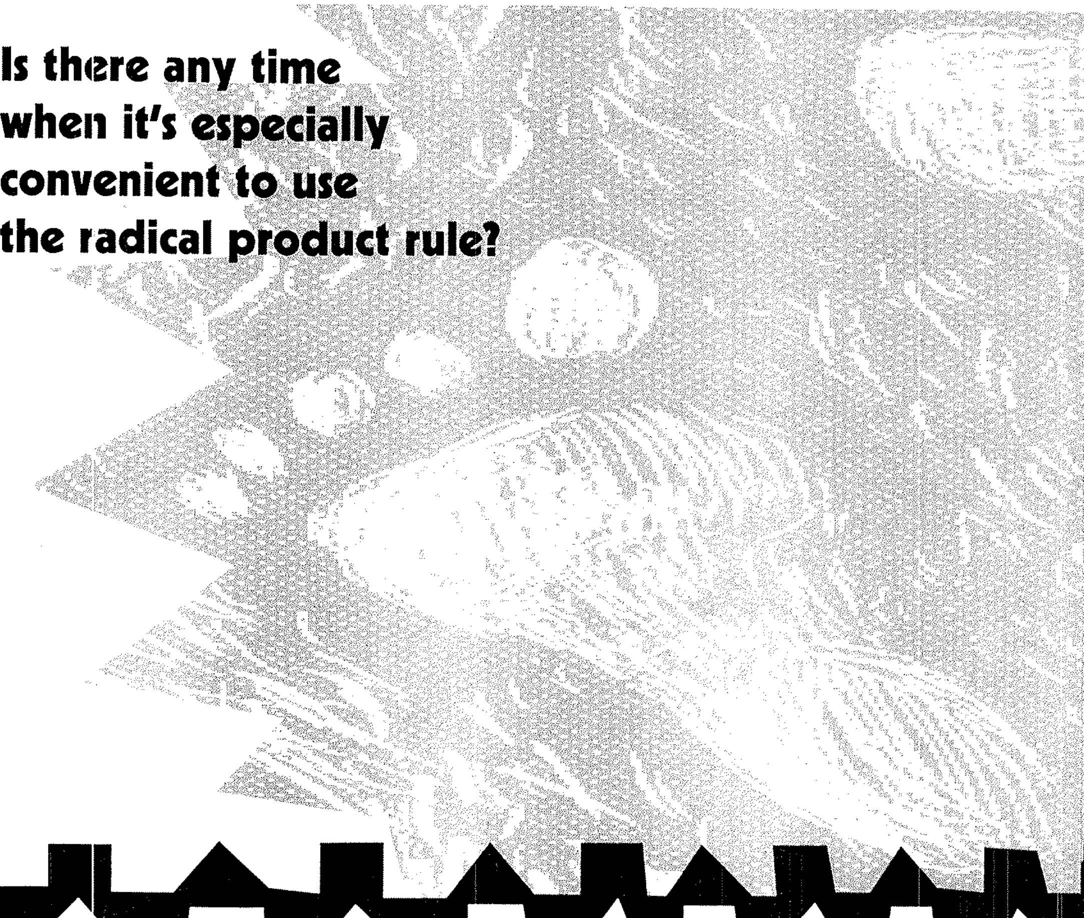
c) $\sqrt{p} \cdot \sqrt{q}$

d) $\sqrt{6} \cdot \sqrt{v} \cdot \sqrt{w}$

e) $\sqrt{3} \cdot \sqrt{7} \cdot \sqrt{r} \cdot \sqrt{u}$

Answers:
a) $\sqrt{6}$
b) $\sqrt{140}$
c) $\sqrt{pq}$
d) $\sqrt{6vw}$
e) $\sqrt{21ru}$

Q: Is there any time when it's especially convenient to use the radical product rule?



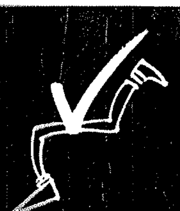
A: Yes. You especially want to use the radical product rule when terms being multiplied give you a perfect square. When that's the case, you can get a nice, neat answer by reducing the perfect square. To your right you'll find the steps showing how to do this.

Simplify:

A) $\sqrt{2} \cdot \sqrt{8}$,

B) $\sqrt{4a} \cdot \sqrt{a}$

Steps	Example A	Example B
1st) Study the terms under the radical sign to see if, when multiplied, they would give you a perfect square.	$\sqrt{2} \cdot \sqrt{8}$ ($2 \cdot 8 = 16$; and **16** is a perfect square)	$\sqrt{4a} \cdot \sqrt{a}$ ($4a \cdot a = 4a^2$; and $4a^2$ is a perfect square)
2nd) Use the radical product rule (**p. 126**).	$\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 8}$ $= \sqrt{16}$	$\sqrt{4a} \cdot \sqrt{a} = \sqrt{4a^2}$
3rd) Reduce the radical to get your answer.	$\sqrt{16} = 4$	$\sqrt{4a^2} = 2a$



Now try simplifying these expressions using the radical product rule:

a) $\sqrt{18} \cdot \sqrt{2}$

b) $\sqrt{32} \cdot \sqrt{2}$

c) $\sqrt{20x} \cdot \sqrt{5x}$

d) $\sqrt{8} \cdot \sqrt{18}$

e) $\sqrt{5m} \cdot \sqrt{45m}$

Answers:
a) 6 **d)** 12
b) 8 **e)** 15m
c) 10x

Q: Can I also use the radical product rule backwards? In other words, can I say this:

$$\sqrt{a \cdot c} = \sqrt{a} \cdot \sqrt{c}$$

A: Yes, thanks to the symmetric property (**p. 10**), you can flip terms around the equal sign. This rule is called the **reverse radical product rule**. It's also called **splitting the square** because you can imagine that you're taking an axe and splitting the radical apart. Notice that you can split the radical even if it contains more than two terms.

Examples of the reverse radical product rule

$$\sqrt{m \cdot v} = \sqrt{m} \cdot \sqrt{v}$$

$$\sqrt{e \cdot m \cdot v} = \sqrt{e} \cdot \sqrt{m} \cdot \sqrt{v}$$

$$\sqrt{3 \cdot m} = \sqrt{3} \cdot \sqrt{m}$$

$$\sqrt{9 \cdot r^2 \cdot u} = \sqrt{9} \cdot \sqrt{r^2} \cdot \sqrt{u}$$

$$\sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3}$$

Take out your axe and split these squares:

a) $\sqrt{r \cdot s}$
b) $\sqrt{9 \cdot 7}$
c) $\sqrt{16 \cdot v}$
d) $\sqrt{36 \cdot x^2}$
e) $\sqrt{50 \cdot y^2 \cdot z^3}$

Answers:

a) $\sqrt{r} \cdot \sqrt{s}$
b) $\sqrt{9} \cdot \sqrt{7}$
c) $\sqrt{16} \cdot \sqrt{v}$
d) $\sqrt{36} \cdot \sqrt{x^2}$
e) $\sqrt{50} \cdot \sqrt{y^2} \cdot \sqrt{z^3}$

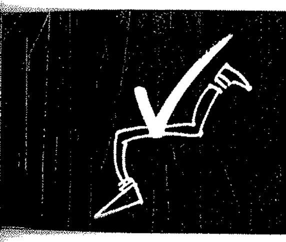
Q: How do I use the reverse radical product rule in a problem?

A: You use the reverse radical product rule to simplify a radical. In other words, you use it to boil a radical term down to a simpler form. Follow the steps shown to your right.

Simplify:

A) $\sqrt{75}$, **B)** $\sqrt{a^2c}$

Steps	Example A	Example B
1st) See if the term under the radical has a factor which is a perfect square. If it does, rewrite the term to show this perfect square.	$\sqrt{75}$ $= \sqrt{25 \cdot 3}$ (**25** is a perfect square)	$\sqrt{a^2c}$ $= \sqrt{a^2 \cdot c}$ (a^2 is a perfect square)
2nd) Take out your axe and split the radical.	$= \sqrt{25} \cdot \sqrt{3}$	$= \sqrt{a^2} \cdot \sqrt{c}$
3rd) Simplify the square root of the perfect square.	$= 5 \cdot \sqrt{3}$ $= 5\sqrt{3}$	$= a \cdot \sqrt{c}$ $= a\sqrt{c}$



Now try simplifying these terms:

a) $\sqrt{8}$ **d)** $\sqrt{100a^2}$
b) $\sqrt{50}$ **e)** $\sqrt{c^2e^2n}$
c) $\sqrt{48}$

Answers:
a) $2\sqrt{2}$ **d)** $10a$
b) $5\sqrt{2}$ **e)** $ce\sqrt{n}$
c) $4\sqrt{3}$

Q: **Now I grasp the basic idea behind the radical product rules, but this makes me wonder about something:**

If $\sqrt{a \cdot c} = \sqrt{a} \cdot \sqrt{c}$,
wouldn't it also be true that
$\sqrt{a + c} = \sqrt{a} + \sqrt{c}$,
and that
$\sqrt{a - c} = \sqrt{a} - \sqrt{c}$?

A: **Not a chance. This is what you call "creative, wrong thinking."**

Neither $\sqrt{a + c}$ nor $\sqrt{a - c}$ can be simplified in any special way.
But maybe you need to see this to believe it. Let's check it by testing whether $\sqrt{16 + 9} = \sqrt{16} + \sqrt{9}$ and also whether $\sqrt{169 - 144} = \sqrt{169} - \sqrt{144}$

$$\sqrt{16 + 9} = \sqrt{16} + \sqrt{9}$$
$$\sqrt{25} = 4 + 3$$
$$5 = 7$$

nope!

$$\sqrt{169 - 144} = \sqrt{169} - \sqrt{144}$$
$$\sqrt{25} = 13 - 12$$
$$5 = 1$$

nope!

So remember: $\sqrt{a + c} \neq \sqrt{a} + \sqrt{c}$

and $\sqrt{a - c} \neq \sqrt{a} - \sqrt{c}$

True or false?

a) $\sqrt{e \cdot r} = \sqrt{e} \cdot \sqrt{r}$
b) $\sqrt{r + u} = \sqrt{r} \cdot \sqrt{u}$
c) $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$
d) $\sqrt{5mr} = \sqrt{5} \cdot \sqrt{m} \cdot \sqrt{r}$
e) $\sqrt{9 - 3} = \sqrt{9} - \sqrt{3}$

Answers:
a) true d) true
b) false e) false
c) false

Q: What would be the value of a term like this:

$$\sqrt{a^2}$$

A: The value of this term is so simple that it's confusing.

$$\sqrt{a^2} = a$$

See below if you want to know why this is true.

From the definition of an exponent (**p. 86**) you know that: $\sqrt{a^2} = \sqrt{a \cdot a}$ (since $a^2 = a \cdot a$). And when you take your axe to the square root, you get: $\sqrt{a \cdot a} = \sqrt{a} \cdot \sqrt{a}$. Then, by the very definition of a square root (**p. 116**), you know that: $\sqrt{a} \cdot \sqrt{a} = a$. Now, if you look at these three statements, you'll see that everything is equal to everything else, so with a grand use of the transitive property (**p. 11**), you say that the first is equal to the last. That is: $\sqrt{a^2} = a$. To the right are more examples.

Examples

$$\sqrt{5^2} = 5$$

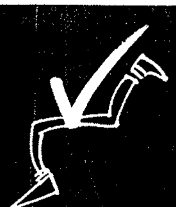
$$\sqrt{y^2} = y$$

$$\sqrt{(5y)^2} = 5y$$

$$\sqrt{(mn)^2} = mn$$

Silly example

$$\sqrt{(\text{popcorn})^2} = \text{popcorn}$$



Simplify these terms:

a) $\sqrt{11^2}$
b) $\sqrt{r^2}$
c) $\sqrt{(7c)^2}$
d) $\sqrt{(\text{frog})^2}$
e) $\sqrt{(12x^2)^2}$

Answers:
a) 11 d) frog
b) r e) $12x^2$
c) 7c

Q: Now I understand how to multiply two radicals together, but how would I multiply terms with numbers both inside and outside radical signs? In other words, how would I work out the answer to something like this:

$$2\sqrt{3} \cdot 5\sqrt{7}$$

A: The rule is this: you multiply the terms **outside** the radical and keep that answer **outside the radical sign**; then you multiply the terms **inside** the radical and keep that answer **inside the radical sign**. The general form for the solution is this:

$$a\sqrt{c} \cdot x\sqrt{n} = a \cdot x\sqrt{c \cdot n}$$

This rule is called the **complex radical product rule**, "complex" meaning "complicated." To the right are examples showing how you'd use the **complex radical product rule**.

Examples

$$m\sqrt{v} \cdot r\sqrt{u} = mr\sqrt{vu}$$

$$m\sqrt{2} \cdot r\sqrt{5} = mr\sqrt{10}$$

$$2\sqrt{m} \cdot 5\sqrt{r} = 10\sqrt{mr}$$

$$2\sqrt{3} \cdot 5\sqrt{7} = 10\sqrt{21}$$

Work out these products using the complex radical product rule:

a) $5\sqrt{2} \cdot 7\sqrt{3}$
b) $a\sqrt{6} \cdot b\sqrt{11}$
c) $8\sqrt{m} \cdot 10\sqrt{n}$
d) $x\sqrt{v} \cdot r\sqrt{u}$
e) $12\sqrt{ax} \cdot w\sqrt{13}$

Answers:
a) $35\sqrt{6}$
b) $ab\sqrt{66}$
c) $80\sqrt{mn}$
d) $xr\sqrt{vu}$
e) $12w\sqrt{13ax}$