Q: Sometimes I see a number outside a radical multiplying a term inside a radical, and this entire expression is itself squared. Is there any shortcut for working out this kind of term, a term like:

$$\left(5\sqrt{3}\right)^2$$

A: Yes, there's a shortcut, and it's this:

$$\left(5\sqrt{3}\right)^2 = 5^2 \cdot 3 = 75$$

Or, in general:

$$\left(a\sqrt{c}\right)^2 = a^2 \cdot c$$

You can remember this as the **radical product shortcut**. To the right are examples showing how you'd use this wonderful little shortcut.

Examples

$$\left(m\sqrt{v}\right)^2 = m^2 \cdot v$$

$$\left(v\sqrt{5}\right)^2 = v^2 \cdot 5 = 5v^2$$

$$\left(3\sqrt{m}\right)^2 = 3^2 \cdot m = 9m$$

$$\left(3\sqrt{7}\right)^2 = 3^2 \cdot 7 = 9 \cdot 7 = 63$$

Use the radical product shortcut to simplify these terms:

a) $\left(3\sqrt{5}\right)^2$ d) $\left(p\sqrt{q}\right)^2$

b) $\left(8\sqrt{a}\right)^2$ e) $\left(3x\sqrt{4y}\right)^2$

c) $\left(c\sqrt{7}\right)^2$

Answers:
a) 45 d) p^2q
b) $64a$ e) $36x^2y$
c) $7c^2$

Q: What does the radical quotient rule say?

$$\frac{\sqrt{a}}{\sqrt{c}} = ?$$

A: The radical quotient rule works just like the radical product rule, except that it deals with a quotient instead of with a product. Here's what it says:

$$\frac{\sqrt{a}}{\sqrt{c}} = \sqrt{\frac{a}{c}}$$

Or in English: **if a fraction has a radical over a radical, you can combine the two separate radicals into one big, happy radical.** To the right are examples showing how you'd use the radical quotient rule.

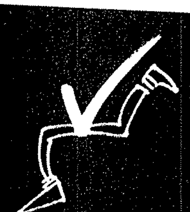
Examples

$$\frac{\sqrt{m}}{\sqrt{v}} = \sqrt{\frac{m}{v}} \qquad \frac{\sqrt{m}}{\sqrt{3}} = \sqrt{\frac{m}{3}}$$

$$\frac{\sqrt{3}}{\sqrt{m}} = \sqrt{\frac{3}{m}} \qquad \frac{\sqrt{25}}{\sqrt{3}} = \sqrt{\frac{25}{3}}$$

Silly example

$$\frac{\sqrt{zig}}{\sqrt{zag}} = \sqrt{\frac{zig}{zag}}$$



Combine using the radical quotient rule:

a) $\frac{\sqrt{v}}{\sqrt{w}}$ c) $\frac{\sqrt{ant}}{\sqrt{fly}}$

b) $\frac{\sqrt{20}}{\sqrt{5}}$ d) $\frac{\sqrt{4a}}{\sqrt{2a}}$

Answers: a) $\sqrt{\frac{v}{w}}$ b) $\sqrt{\frac{20}{5}}$ c) $\sqrt{\frac{ant}{fly}}$ d) $\sqrt{\frac{4a}{2a}}$

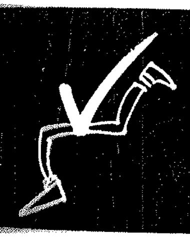
When do I use the radical quotient rule in a problem?

A: You use the radical quotient rule when two or more terms divide to make a perfect square. Follow the steps shown to the right.

Simplify:

A) $\frac{\sqrt{8}}{\sqrt{2}}$, **B)** $\frac{\sqrt{a^3}}{\sqrt{a}}$

Steps	Example A	Example B
1st) Look at the terms under the radical signs to see if, when divided, they make a perfect square.	$\frac{\sqrt{8}}{\sqrt{2}} \Big\} \frac{8}{2} = 4$ (**4** is a perfect square)	$\frac{\sqrt{a^3}}{\sqrt{a}} \Big\} \frac{a^3}{a} = a^2$ (a^2 is a perfect square)
2nd) Use the radical quotient rule (**p. 134**).	$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4}$	$\frac{\sqrt{a^3}}{\sqrt{a}} = \sqrt{\frac{a^3}{a}} = \sqrt{a^2}$
3rd) Reduce radical using definition of a square root (**p. 116**).	$\sqrt{4} = 2$	$\sqrt{a^2} = a$



Simplify using the radical quotient rule:

a) $\frac{\sqrt{50}}{\sqrt{2}}$ c) $\frac{\sqrt{48}}{\sqrt{3}}$

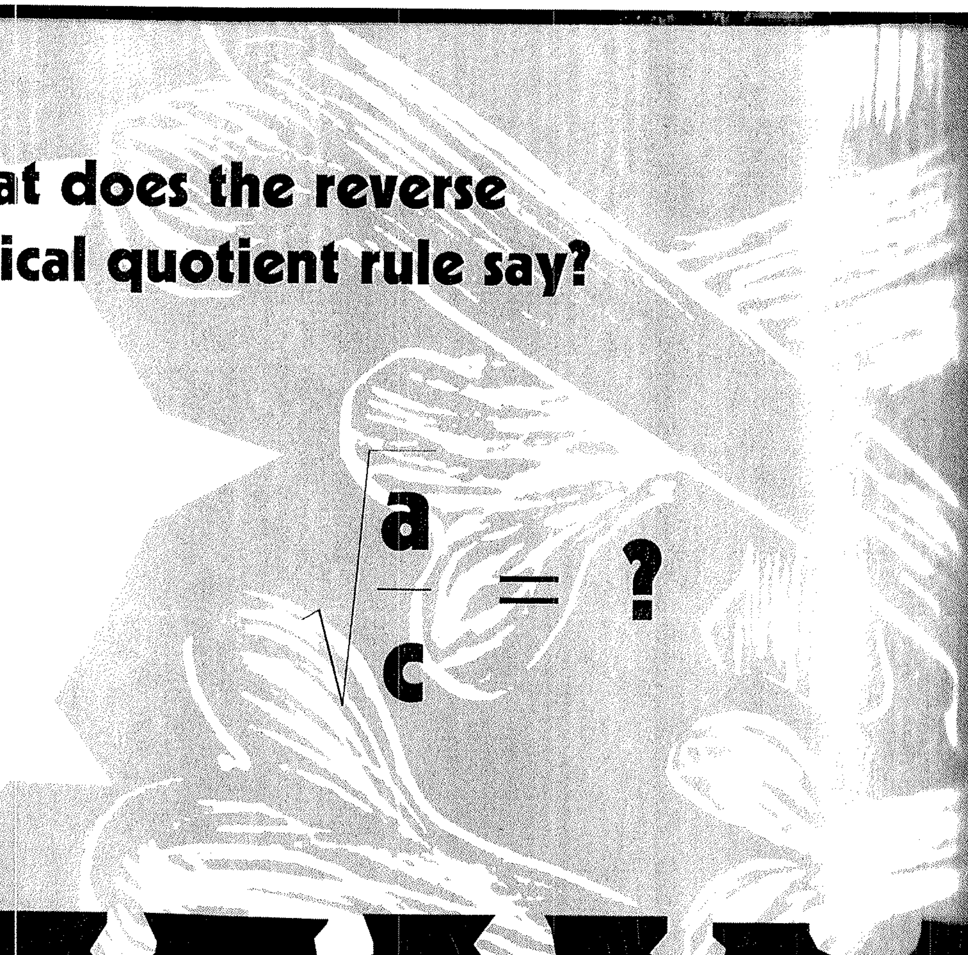
b) $\frac{\sqrt{d^5}}{\sqrt{d^3}}$ d) $\frac{\sqrt{f^8}}{\sqrt{f^4}}$

Answers:

a) 5 **c)** 4

b) d **d)** f^2

Q: What does the reverse radical quotient rule say?



$$\sqrt{\frac{a}{c}} = ?$$

A: The reverse radical quotient rule says more or less what you'd expect it to say, namely this:

$$\sqrt{\frac{a}{c}} = \frac{\sqrt{a}}{\sqrt{c}}$$

Like the reverse radical product rule, the reverse radical quotient rule involves **splitting the square**, only here you take your axe and swing at the radical sideways. To the right are examples showing how you'd use the reverse radical quotient rule.

Examples

$$\sqrt{\frac{m}{v}} = \frac{\sqrt{m}}{\sqrt{v}} \qquad \sqrt{\frac{m}{3}} = \frac{\sqrt{m}}{\sqrt{3}}$$

$$\sqrt{\frac{3}{m}} = \frac{\sqrt{3}}{\sqrt{m}} \qquad \sqrt{\frac{25}{3}} = \frac{\sqrt{25}}{\sqrt{3}}$$

Silly example

$$\sqrt{\frac{\text{peach}}{\text{pear}}} = \frac{\sqrt{\text{peach}}}{\sqrt{\text{pear}}}$$

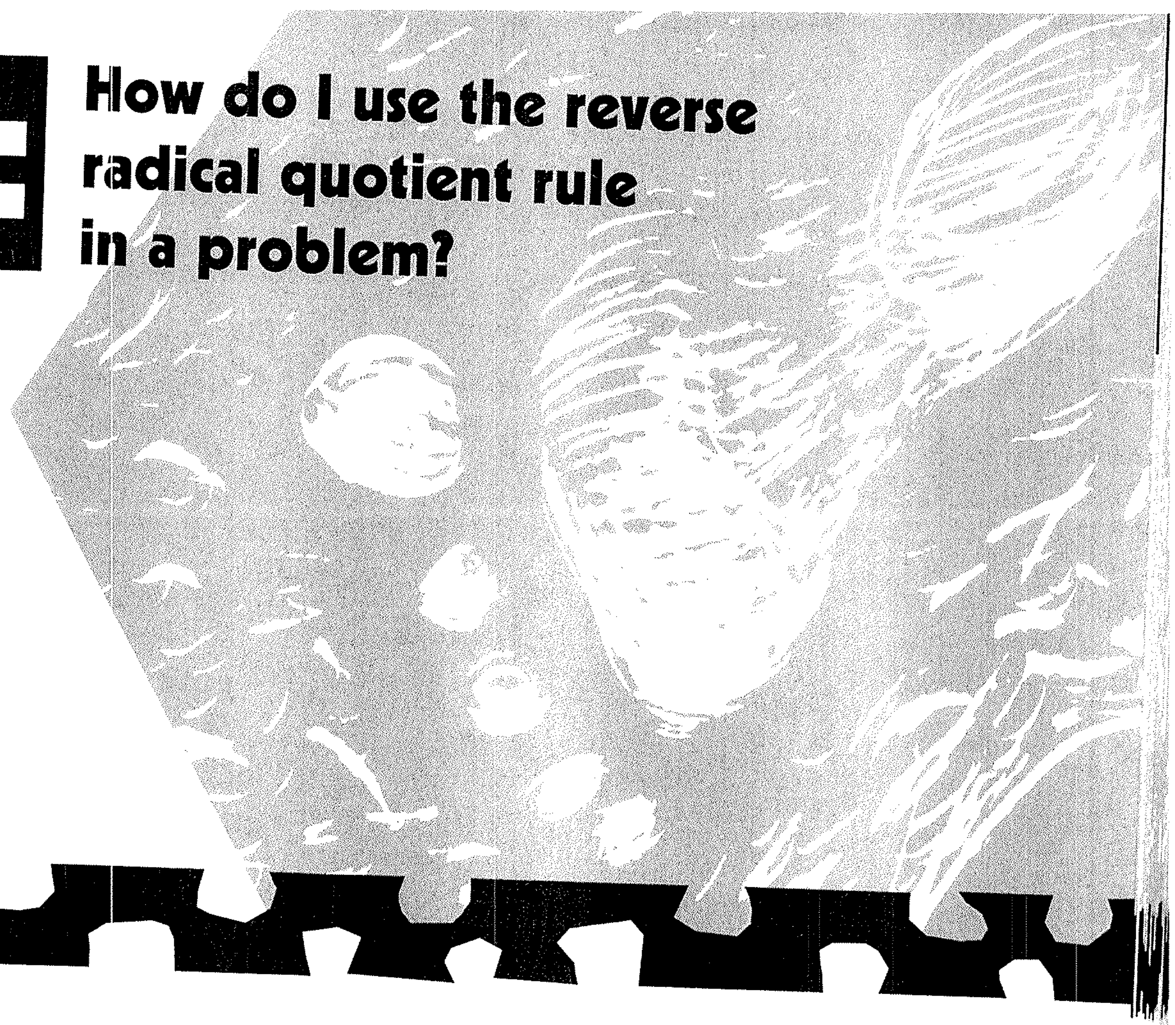
Now take out your axe and split these radicals using the reverse radical quotient rule:

a) $\sqrt{\frac{x}{y}}$ b) $\sqrt{\frac{2}{9}}$ c) $\sqrt{\frac{p}{7}}$ d) $\sqrt{\frac{\text{elm}}{\text{oak}}}$

Answers:

a) $\frac{\sqrt{x}}{\sqrt{y}}$ b) $\frac{\sqrt{2}}{\sqrt{9}}$ c) $\frac{\sqrt{p}}{\sqrt{7}}$ d) $\frac{\sqrt{\text{elm}}}{\sqrt{\text{oak}}}$

Q: How do I use the reverse radical quotient rule in a problem?



A: You use the reverse radical quotient rule to simplify a fraction under a radical. Follow the steps shown to the right.

Simplify:

A) $\sqrt{\frac{7}{25}}$,

B) $\sqrt{\frac{a}{c^2}}$

Steps	Example A	Example B
1st) See if either the numerator or the denominator contains a perfect square.	$\sqrt{\frac{7}{25}}$ (**25** is a perfect square)	$\sqrt{\frac{a}{c^2}}$ (c^2 is a perfect square)
2nd) If it does, take out your axe and split that radical.	$\sqrt{\frac{7}{25}} = \frac{\sqrt{7}}{\sqrt{25}}$	$\sqrt{\frac{a}{c^2}} = \frac{\sqrt{a}}{\sqrt{c^2}}$
3rd) Simplify the square root of the perfect square.	$= \frac{\sqrt{7}}{5}$	$= \frac{\sqrt{a}}{c}$

Simplify using the reverse radical quotient rule:

a) $\sqrt{\frac{v}{49}}$ c) $\sqrt{\frac{r^2}{s^2}}$

b) $\sqrt{\frac{11}{x^2}}$ d) $\sqrt{\frac{16}{49}}$

Answers: a) $\frac{\sqrt{v}}{7}$ b) $\frac{\sqrt{11}}{x}$ c) $\frac{r}{s}$ d) $\frac{4}{7}$

Q: Now I have some sense of how to work with radicals. But what do I do when I have a fraction, and both its numerator and its denominator have one term inside a radical and another term outside a radical. In other words, how would I simplify an expression like this:

$$\frac{6\sqrt{18}}{9\sqrt{2}}$$

A: The rule for doing this, called the **complex radical quotient rule**, works just like the complex radical product rule (**p. 132**), except that here you divide instead of multiply. You divide the terms **outside** the radical and keep that answer **outside the radical sign**; then you divide the terms **inside** the radical and keep that answer **inside the radical**. The general form for the solution looks like this:

$$\frac{a\sqrt{c}}{x\sqrt{n}} = \frac{a}{x}\sqrt{\frac{c}{n}}$$

To the right are examples showing how you use the complex radical quotient rule.

Examples

$$\frac{m\sqrt{v}}{r\sqrt{u}} = \frac{m}{r}\sqrt{\frac{v}{u}} \qquad \frac{3\sqrt{m}}{5\sqrt{r}} = \frac{3}{5}\sqrt{\frac{m}{r}}$$

$$\frac{m\sqrt{3}}{r\sqrt{5}} = \frac{m}{r}\sqrt{\frac{3}{5}} \qquad \frac{3\sqrt{6}}{5\sqrt{7}} = \frac{3}{5}\sqrt{\frac{6}{7}}$$

Simplify using the complex radical quotient rule:

a) $\frac{c\sqrt{e}}{n\sqrt{w}}$

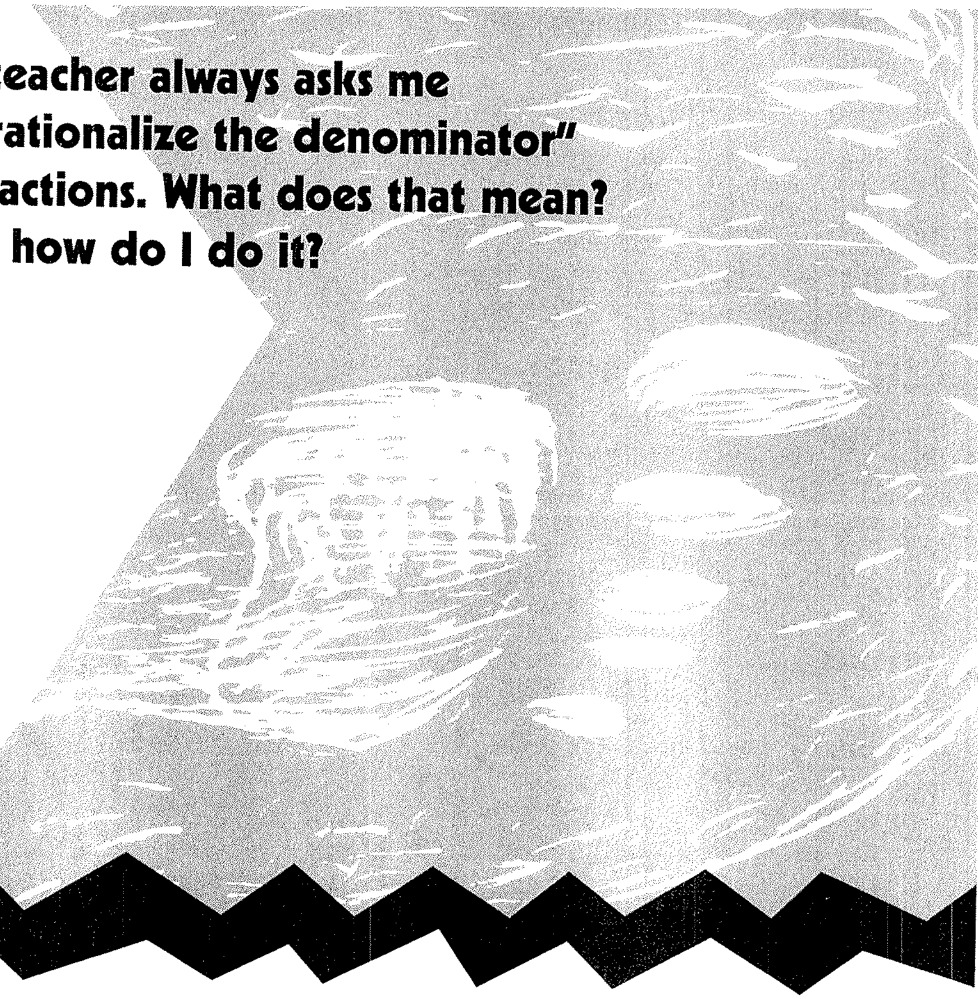
b) $\frac{s\sqrt{12}}{x\sqrt{5}}$

c) $\frac{6\sqrt{y}}{8\sqrt{z}}$

d) $\frac{2\sqrt{8}}{3\sqrt{11}}$

Answers: a) $\frac{c}{n}\sqrt{\frac{e}{w}}$ b) $\frac{s}{x}\sqrt{\frac{12}{5}}$ c) $\frac{3}{4}\sqrt{\frac{y}{z}}$ d) $\frac{2}{3}\sqrt{\frac{8}{11}}$

Q: My teacher always asks me to "rationalize the denominator" of fractions. What does that mean? And how do I do it?



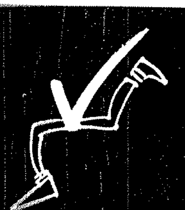
A: Rationalizing the denominator means playing around with the radical in the denominator to turn it into a whole number or a rational term. Just follow the steps to the right.

Rationalize the denominator of the fraction:

$$\frac{\sqrt{5}}{\sqrt{18}}$$

Steps	**Example**
1st) If you can simplify the denominator, do so using the reverse radical product rule (**p. 129**).	$\frac{\sqrt{5}}{\sqrt{18}} = \frac{\sqrt{5}}{3\sqrt{2}}$
2nd) Multiply numerator and denominator by the radical in the denominator.	$\frac{\sqrt{5}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
3rd) Simplify numerator by using radical product rule (**p. 126**), if necessary. Simplify denominator by using definition of a square root (**p. 116**).	$\frac{\sqrt{5}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{3 \cdot 2} = \frac{\sqrt{10}}{6}$

In Step 1, you don't **always** have to simplify the denominator. It may already be in simplest radical form. For example, in $\frac{\sqrt{2}}{\sqrt{3}}$, the denominator, $\sqrt{3}$, already is in simplest radical form. So here you would only need to multiply the numerator and denominator by $\sqrt{3}$.



Now try rationalizing the denominators of these fractions:

a) $\frac{\sqrt{2}}{\sqrt{3}}$ **c)** $\frac{\sqrt{3}}{\sqrt{8}}$

b) $\frac{5}{\sqrt{7}}$ **d)** $\frac{11}{\sqrt{27}}$

Answers: a) $\frac{\sqrt{6}}{3}$ b) $\frac{5\sqrt{7}}{7}$ c) $\frac{\sqrt{6}}{4}$ d) $\frac{11\sqrt{3}}{9}$