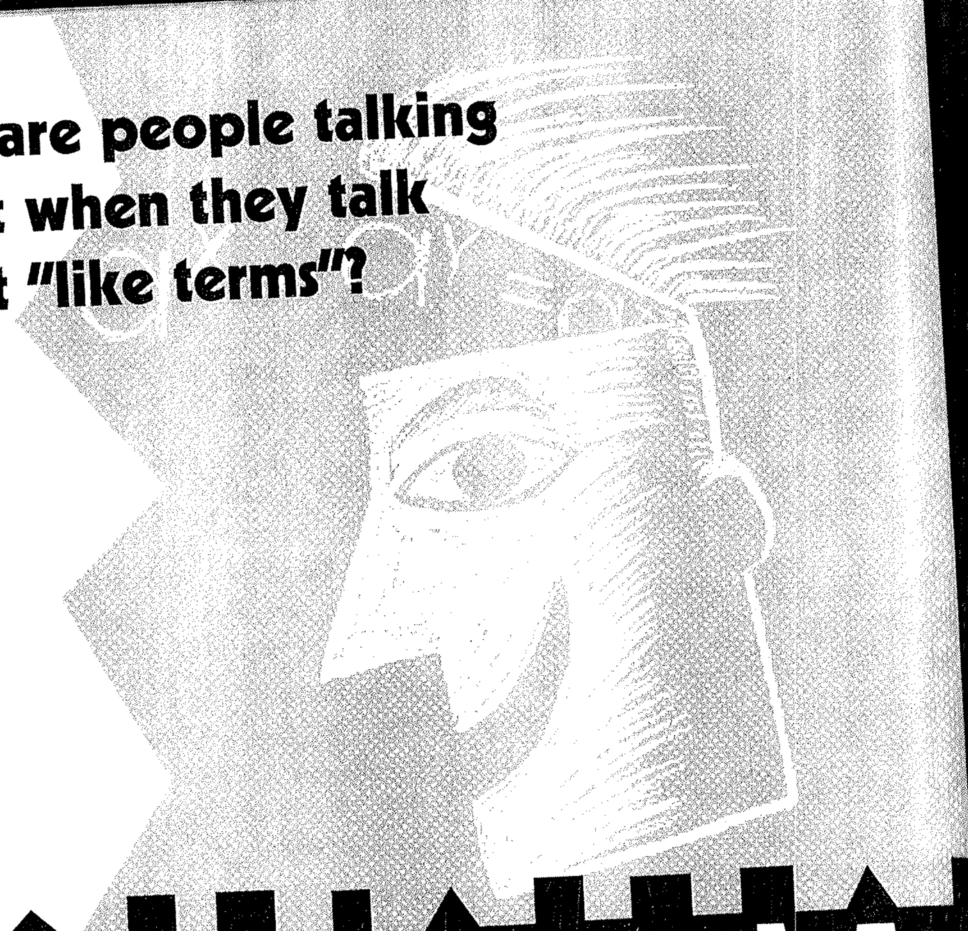
Q: What are people talking about when they talk about "like terms"?

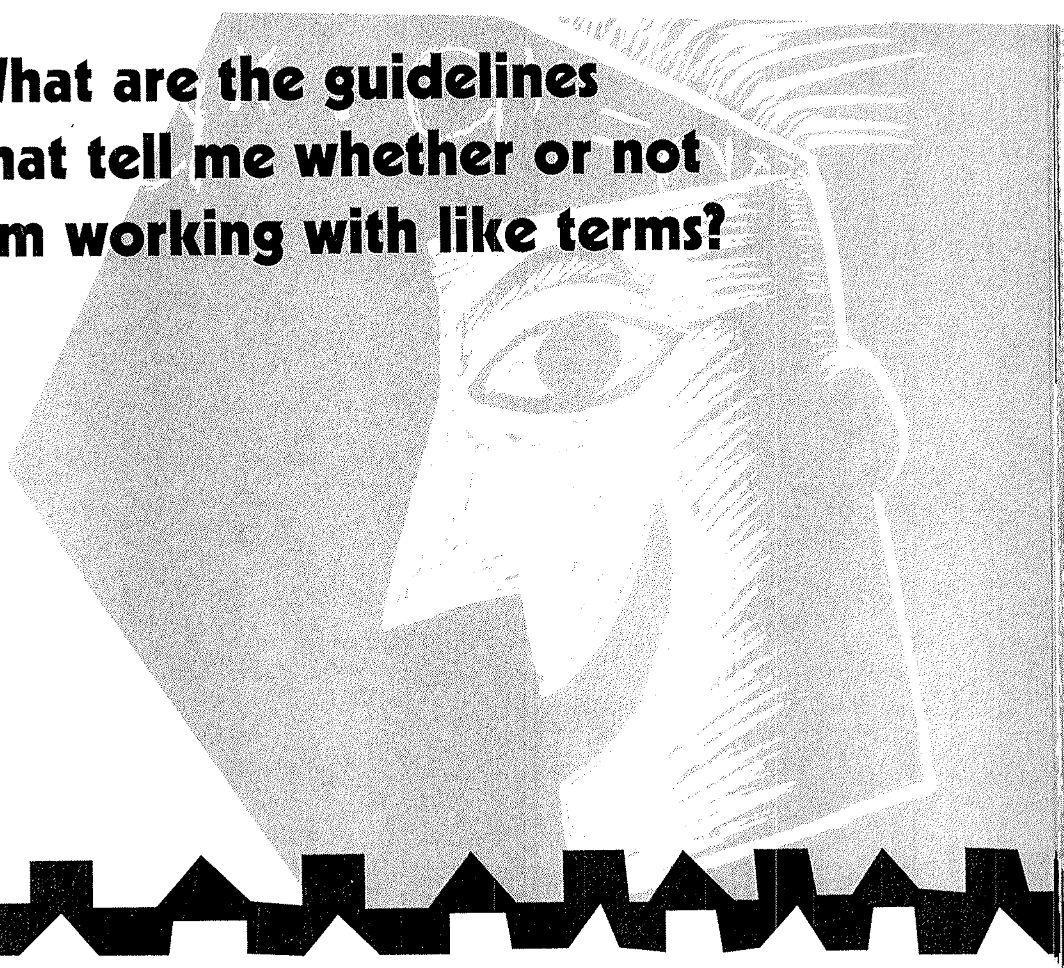


A: "Like terms" is math-talk for terms that represent the same kinds of things. Since like terms stand for the same kinds of things, you're allowed to add them to each other and subtract them from each other. This is virtually impossible to understand without examples, so here are a few:

Like terms	Why are these like terms?
2, – 4, 3, 0, 1/ 7	All are number terms.
2a, – 3a, a/2	All are **a** terms.
ac, ca, – 3ac	All are terms with **a** and **c**.
acm, mca, 4mca	All are terms with **a**, **c** and **m**.
a^2m, ma^2, aam	All are terms with two **a**'s and one **m**.

Unlike terms	Why are these unlike terms?
3, y	First is a number term; second is a **y** term.
4, 2a	First is a number term; second is an **a** term.
2a, 2c	First is an **a** term; second is a **c** term.
4acm, 3ac	First is a term with **a**, **c** and **m**; second is a term with just **a** and **c**.
a^2m, am^2	First has two **a**'s and one **m**; second has two **m**'s and one **a**.

Q: What are the guidelines that tell me whether or not I'm working with like terms?



A: Look below. These three simple guidelines will let you know at a glance.

a) All numbers are like terms.

Examples

2, − **7**, **1/3**, − **.62**, and **478** are all like terms

b) All terms with the same combination of variables are like terms, whether or not the variables are in the same order.

Examples

ax^2 and x^2a are like terms

ac and **ca** are like terms

c) A variable term standing alone and the same term with a number in front are like terms.

Examples

ac and **2ac** are like terms;

4xma and **axm** are like terms

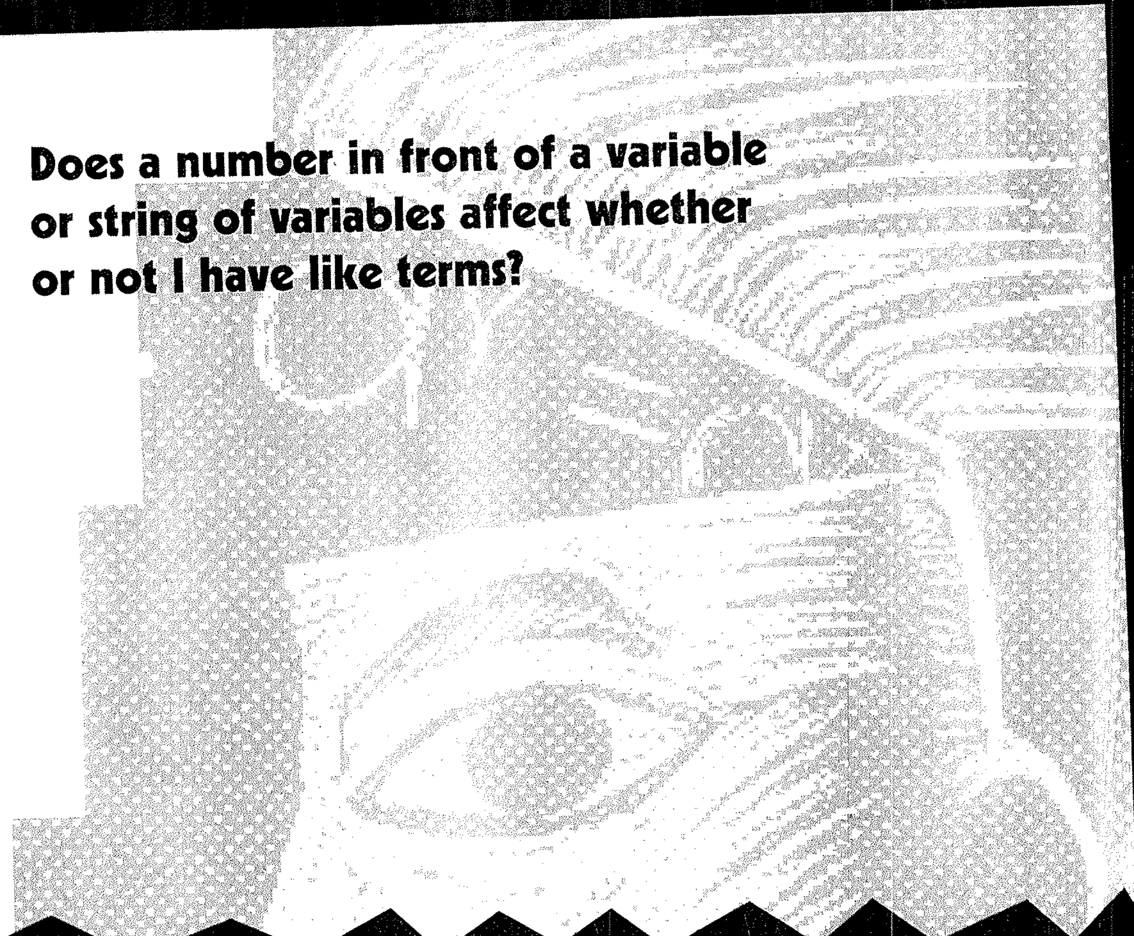
Note: whenever a variable term has no number before it, be aware that there's actually an invisible **1** before it.

That is, **ac** means **1ac**; x^2y means $1x^2y$, etc.

Tell whether or not these terms are like terms:

a) − 3, .49, 4/11
b) xy, 7xy, 300yx
c) 12mrp, 6pr, 9mr
d) p^2q^2, q^2p^2, 7qp
e) fgh, ghf, hgf

Answers:
a) like terms
b) like terms
c) not like terms
d) not like terms
e) like terms



Q: Does a number in front of a variable or string of variables affect whether or not I have like terms?

A: No. The number simply tells you how many of that term you have.

Example

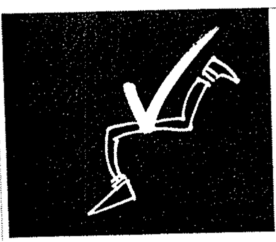
7ac and **ac** are like terms.

ac means **1ac**, while **7ac** means **7ac**'s

When you do simple addition and subtraction with such terms, this becomes more clear.

$$7ac + ac = 8ac$$

$$7ac - ac = 6ac$$

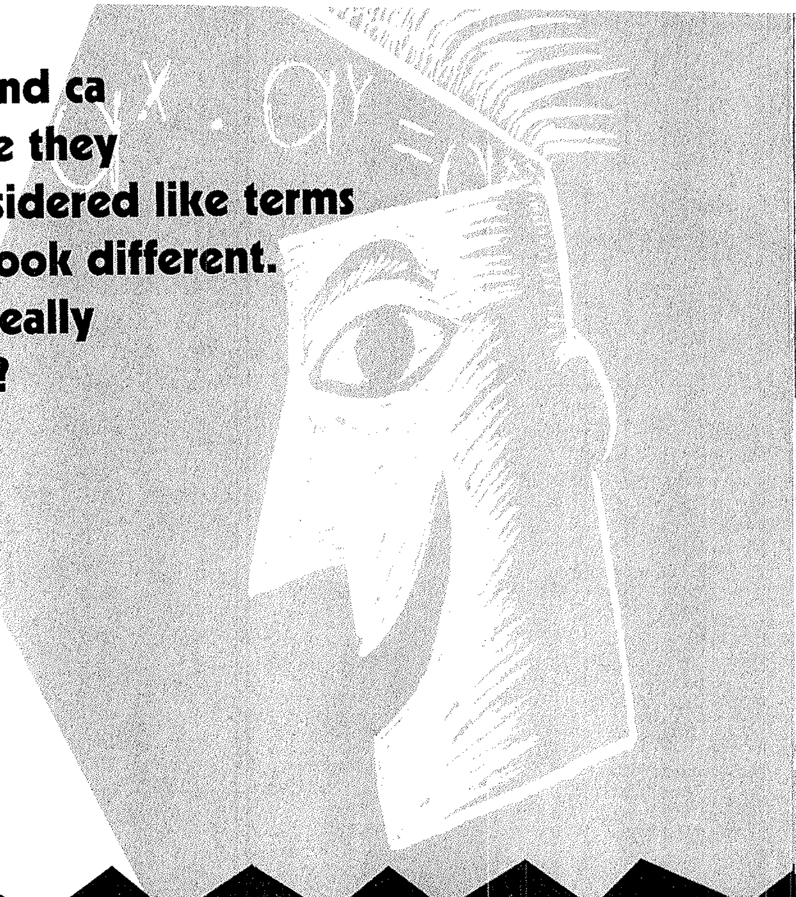


Add or subtract the like terms:

a) 9b + b
b) 13xyz − xyz
c) pq + pq
d) y + y + y + y
e) qrs − qrs

Answers:
a) 10b
b) 12xyz
c) 2pq
d) 4y
e) 0 (meaning " zero")

Q: Terms like ac and ca don't seem like they should be considered like terms because they look different. What guarantees that these really are the same kinds of things?



A: If you think about it, the commutative property (p. 12) guarantees that ac and ca are in fact like terms.

Since the commutative property holds true for multiplication,

$$a \times c = c \times a$$

which means that:

$$ac = ca$$

This means that two strings of variables can be like terms even if the order of letters is switched around.

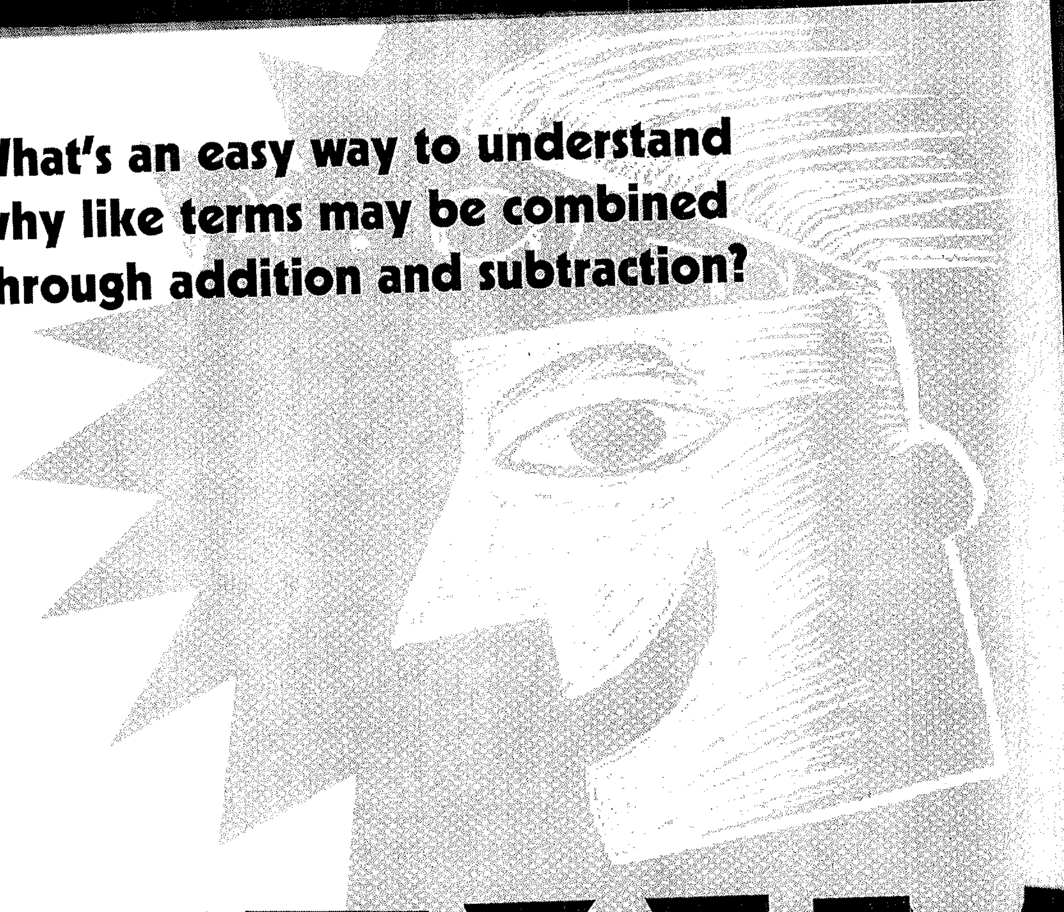
More examples of bizarre-looking like terms

ax^2m **and** mx^2a

xna **and** $7axn$

$5mrv$ **and** $7rvm$

Q: What's an easy way to understand why like terms may be combined through addition and subtraction?



A: If you stretch your mind and think of like terms as things you can touch or hold, the rules for combining them start to make sense.

Most sane people would agree that: **2 cats** + **3 cats** = **5 cats**. Why? Because cats are cats. They're the same kind of thing; therefore they can be combined, or grouped together.

Most sane people also would agree that: **2 cats** + **3 dogs** = **2 cats** + **3 dogs**. Here we don't get a single answer on the right side because cats and dogs are different kinds of things, and therefore they can't be combined.

Now if we just let the letter **c** stands for cat, and the letter **d** stands for dog, we could write the same ideas algebraically, like this:

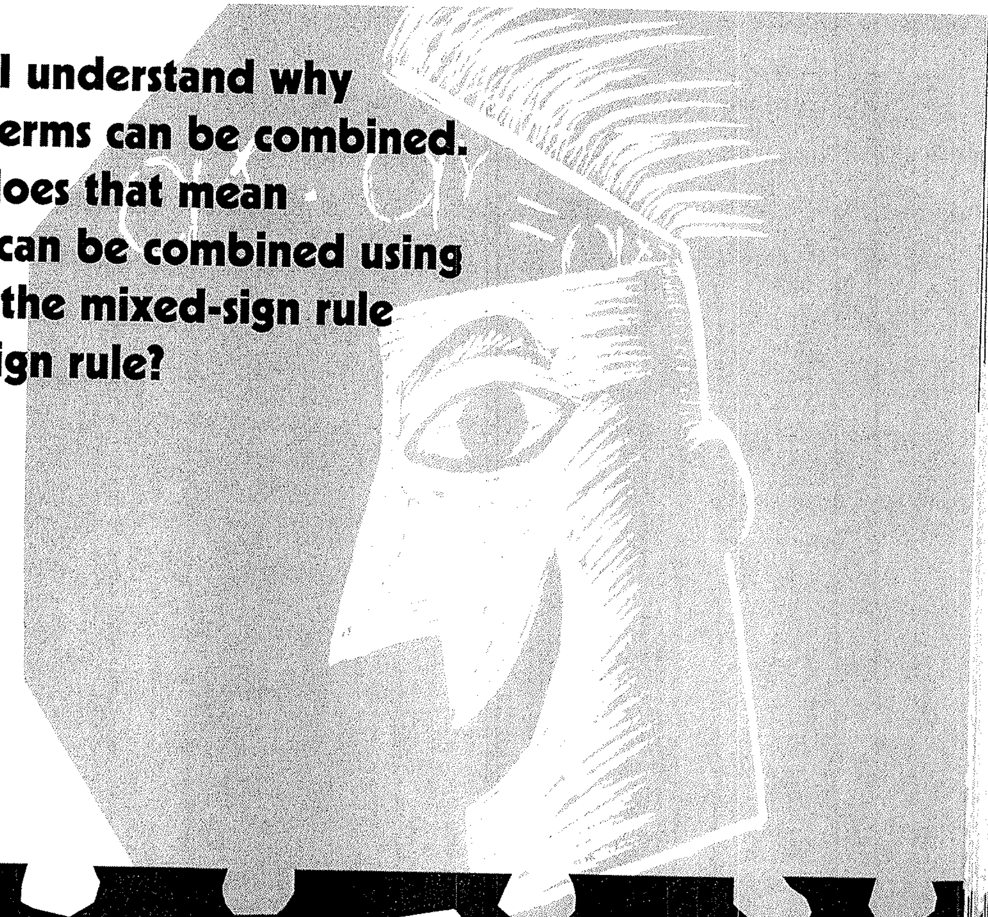
$$2c + 3c = 5c \quad \text{but} \quad 2c + 3d = 2c + 3d$$

Like terms are just like cats or dogs; they are the same kind of thing, and that is why they can be combined.

Tip: be creative — try to think of variables as real things. For example, in a problem like: $4x + 2x$, you might imagine that $4x$ stands for **four eggs**. Then the problem would read: **four eggs** + **two eggs**. Obviously in real life this would equal **six eggs**, so the answer in algebra is $6x$.

(Hmmm ... what real thing could you say "xyz" stands for? Be as creative as you dare.)

Q: Now I understand why like terms can be combined. But does that mean they can be combined using the same-sign rule, the mixed-sign rule and the neighbor-sign rule?



A: Yes, like terms can be combined using all three of these rules.
Below you'll find examples demonstrating this idea.

Same-sign rule

$$-\ 3a - 5a$$
$$= -\left(3a + 5a\right)$$
$$= -\ 8a$$

$$+\ 3a + 5a$$
$$= +\left(3a + 5a\right)$$
$$= +\ 8a$$

Mixed-sign rule

$$+\ 3pq - 8pq$$
$$= -\left(8pq - 3pq\right)$$
$$= -\ 5pq$$

$$-\ 3pq + 8pq$$
$$= +\left(8pq - 3pq\right)$$
$$= +\ 5pq$$

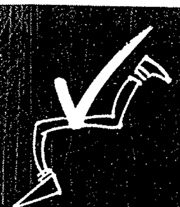
Neighbor-sign rule

$$9v + +\ 5v$$
$$= 9v + 5v$$
$$= 14v$$

$$9v + -\ 5v$$
$$= 9v - 5v$$
$$= 4v$$

$$9v - -\ 5v$$
$$= 9v + 5v$$
$$= 14v$$

$$9v - +\ 5v$$
$$= 9v - 5v$$
$$= 4v$$



Simplify:

a) $-\ 3ab - 4ab$
b) $8x^2 - -\ 2x^2$
c) $-\ 7xyz + 10xyz$
d) $13a^2b + -\ 4a^2b$
e) $+\ pv - 10pv$
f) $+\ 6rs + 11rs$

Answers:
a) $-\ 7ab$ d) $9a^2b$
b) $10x^2$ e) $-\ 9pv$
c) $3xyz$ f) $17rs$

Q: **O.K., now I understand how to combine like terms with the same-, mixed- and neighbor-sign rules. But what do I do when I have various kinds of like terms in the same problem? In other words, how would I simplify something like this:**

$$6v \; - + \; 2xy \; - - \; 8v \; + \; 4xy$$

A: You would use **N**ate's **Gr**eat **S**trawberry **M**ousse once again, as shown to the right.

Simplify:

$6v \; - + \; 2xy \; - - \; 8v \; + \; 4xy$

Steps	Example
	$6v \; - + \; 2xy \; - - \; 8v + 4xy$
1st) Use the **N**eighbor-sign rule (**pp. 43-45**).	$= 6v - 2xy + 8v + 4xy$
2nd) **Gr**oup like terms (**p. 42**).	$= 6v + 8v \quad - 2xy + 4xy$
3rd) Use **S**ame-sign rule (**pp. 36-38**).	$= +(6v + 8v) \quad - 2xy + 4xy$ $= +14v \quad - 2xy + 4xy$
4th) Use **M**ixed-sign rule (**pp. 39-41**).	$= +14v \quad + (4xy - 2xy)$ $= +14v + 2xy$

Try simplifying these expressions:

a) $8a \; + + \; 9c \; - + \; 2a - 5c$

b) $2 \; - - \; 3x + 7 - 4x$

c) $-5rs \; - + \; z - 8z \; + - \; 6rs + 4z$

d) $-1 - x \; + - \; 1 + x \; - - \; 1$

e) $y \; + - \; z \; - + \; z \; - - \; y$

Answers:

a) $6a + 4c$

b) $9 - x$

c) $-11rs - 5z$

d) -1

e) $2y - 2z$

Q: There's still one situation that confuses me. It's when I have a positive or a negative sign in front of a bunch of terms inside an enclosure mark, like a parenthesis. How do I simplify expressions like:

$$+\left(a-7\right)$$

and

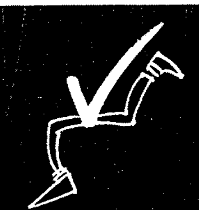
$$-\left(a-7\right)$$

A: Good question, for this is a point that befuddles even the clearest thinkers. The easiest way to simplify these tricky little expressions is to squeeze a tiny **1** between the sign and the parenthesis, and then to multiply using the distributive property. Here are the steps:

Simplify: A) $+(a-7)$, B) $-(a-7)$

Steps	Example A	Example B
1st) Stick a **1** between the sign and the left bracket of the parenthesis.	$+(a-7)$ **becomes** $+1(a-7)$	$-(a-7)$ **becomes** $-1(a-7)$
2nd) Multiply using the distributive property (**pp. 15-16**) to get your answer.	$+1(a-7)$ $=+a-7$	$-1(a-7)$ $=-a+7$

What puzzles people the most is that in **Example B**, the second term of the answer becomes positive. Here's why: when you use the distributive property, you actually multiply the $-$**1** before the parenthesis with the negative term in the parenthesis. And a negative times a negative is a positive (**p. 49**), so in this case the second term is positive.



Now try these:

a) $+(x-9)$
b) $-(x-9)$
c) $+(x+y-z)$
d) $-(x+y-z)$
e) $+(17-m)$

Answers:
a) $x-9$
b) $-x+9$
c) $x+y-z$
d) $-x-y+z$
e) $17-m$

Q: What about situations in which a set of parentheses comes in the midst of a larger problem. How would I simplify problems like these:

$$9 + (a - 7)$$

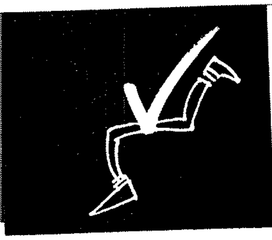
$$9 - (a - 7)$$

A: Just remember the order of operations. It tells you to do multiplication before using the same- or mixed-sign rule.

Here are the steps:

Simplify: A) $9 + (a - 7)$, **B)** $9 - (a - 7)$

Steps	Example A	Example B
1st) Stick in the invisible **1** and multiply using the distributive property (**pp. 15-16**).	$9 + (a - 7)$ $= 9 + 1(a - 7)$ $= 9 + a - 7$	$9 - (a - 7)$ $= 9 - 1(a - 7)$ $= 9 - a + 7$
2nd) Group like terms, then combine terms using the same- or mixed-sign rule (**pp. 36-41**).	$= +9 - 7 + a$ $= +(9 - 7) + a$ $= +2 + a$	$= +9 + 7 - a$ $= +(9 + 7) - a$ $= +16 - a$



Now try these problems:

a) $11 + (c + 4)$
b) $11 - (c + 4)$
c) $13 - (d - 9)$
d) $13 - (d + 9)$
e) $14 - (x + 2)$

Answers:

a) $15 + c$
b) $7 - c$
c) $22 - d$
d) $4 - d$
e) $12 - x$

Q: So if I can combine like terms using the same-, mixed-, and neighbor-sign rules, that means I can also combine like terms using the multiplication and division rules. What I'm saying is:

$$(2a) \cdot (3a) = 6a$$

and

$$8a/4a = 2a$$

Right?

A: No, not in the least! The multiplication and division rules don't work for like terms. Here's why:

Multiplication rule

As you'll see in the Exponents section, a variable times itself does not equal the variable; instead, it equals that variable squared. To give an example,

$$a \cdot a \neq a$$

Instead,

$$a \cdot a = a^2$$

Since that's true,

$$(2a) \cdot (3a) \neq 6a$$

Instead,

$$(2a) \cdot (3a) = 6a^2$$

If you want more details on this, turn to **p. 86**.

Division rule

As you'll see in the Cancelling section, when a variable stands over itself in a fraction, it cancels itself out,and by doing so, it turns into **1**. As an example,

$$a/a \neq a$$

Instead,

$$a/a = 1$$

Since that's true,

$$8a/4a \neq 2a$$

Instead,

$$8\cancel{a}/4\cancel{a} = 8/4 = 2$$

If you want more details on this, turn to **pp. 168-171**.